Afgeleide: rekenregels

www.karelappeltans.be

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Contents

1 Afgeleide machtsfunctie

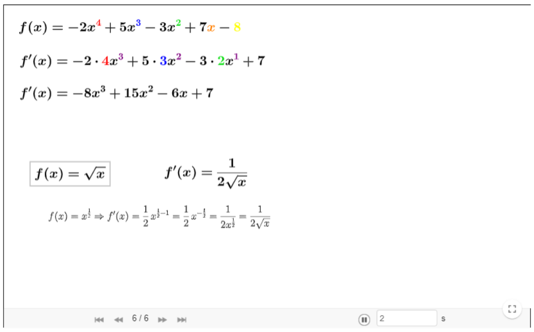


Figure 1: https://www.geogebra.org/m/rkbXbnRv

$$\boxed{(x^n)' = nx^{n-1}}$$

2 Afgeleide rationale functies

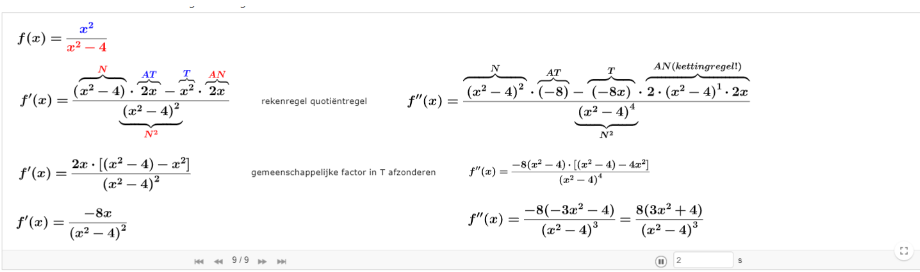


Figure 2: https://www.geogebra.org/m/MpFEGPfT

$$\boxed{\left(\frac{T}{N}\right)' = \frac{NAT - TAN}{N^2}}$$

3 Afgeleide irrationale functies

$$\boxed{(\sqrt{x})' = \frac{1}{2\sqrt{x}}}$$

4 Afgeleide goniometrische functies

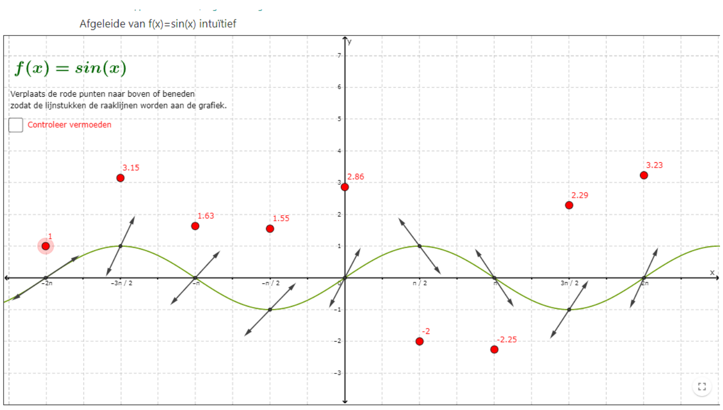


Figure 3: https://www.geogebra.org/m/qkq5rdwr

$$\boxed{(\sin(x))'=\cos(x)}$$

$$\boxed{(\cos(x))'=-\sin(x)}$$

$$\boxed{(\tan(x))'=1\frac{}{\cos^2(x)}}$$

5 Algemene rekenregels

5.1 De somregel

$$\boxed{(a \cdot f(x) + b \cdot g(x))' = a \cdot f'(x) + b \cdot g'(x)}$$

$$(3sin(x) + 5\sqrt{x})' = 3cos(x) + \frac{5}{2\sqrt{x}}$$

5.2 De productregel

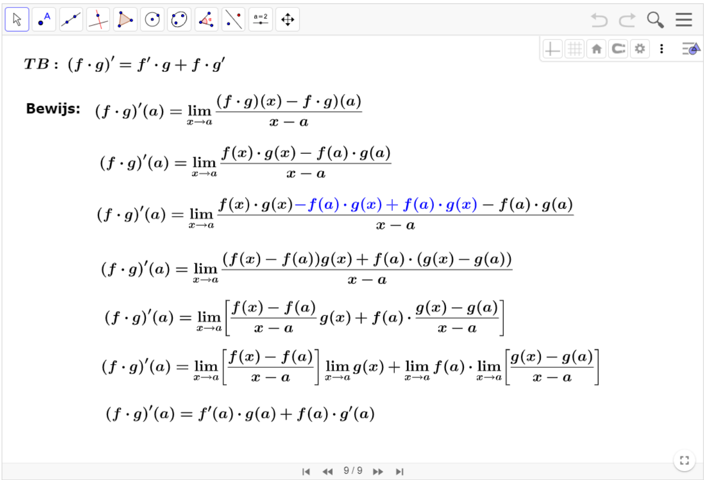


Figure 4: https://www.geogebra.org/m/yvrncth8

$$\boxed{(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)}$$

$$\left(x^2 \cdot cos(x)\right)' = 2x \cdot cos(x) + x^2 \cdot (-sin(x))$$

5.3 De quotiëntregel

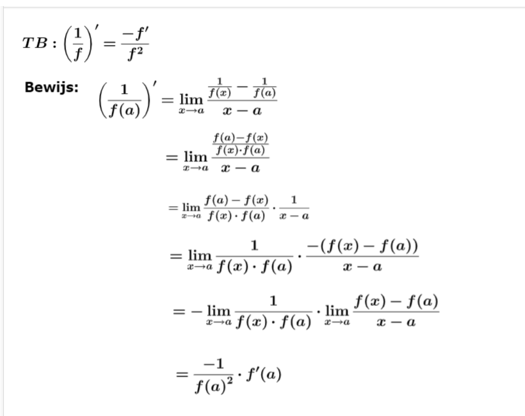


Figure 5: https://www.geogebra.org/m/yvrncth8

$$\boxed{\left(\frac{1}{f(x)}\right)' = \frac{-f'(x)}{f(x)^2}}$$

$$\left(\frac{1}{sin(x)}\right)' = \frac{-cos(x)}{sin^2(x)}$$

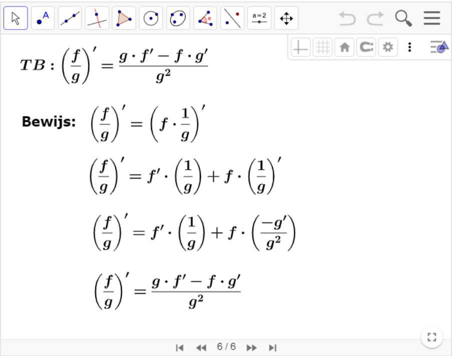


Figure 6: https://www.geogebra.org/m/yvrncth8

$$\boxed{\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{g(x)^2}}$$

$$\left(\frac{2x^3 + 4x^2 + 1}{sin(x)}\right)' = \frac{sin(x) \cdot (6x^2 + 8x) - (2x^3 + 4x^2 + 1) \cdot cos(x)}{sin^2(x)}$$

5.4 De kettingregel

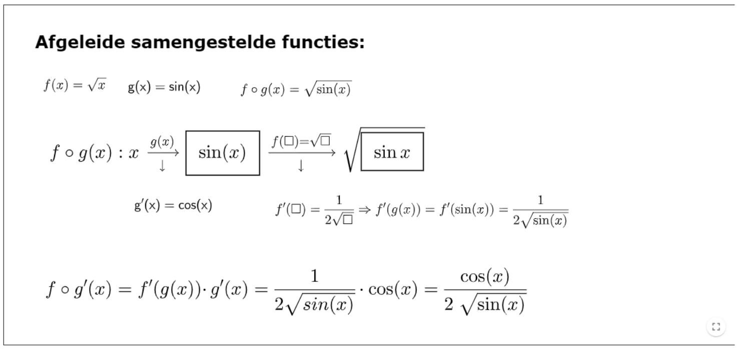


Figure 7: https://www.geogebra.org/m/YN8wHwSh

$$\boxed{(f(g(x))' = f'(g(x)) \cdot g'(x)}$$

$$\left(sin(x^2 + 3)\right)' = cos(x^2 + 3) \cdot 2x$$

5.5 Afgeleide inverse functie

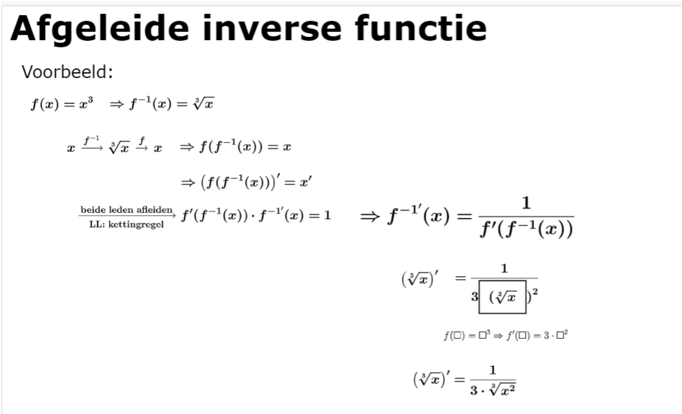


Figure 8: https://www.geogebra.org/m/hwzhaqsb

$$\boxed{\left(f^{-1}(x)\right)' = \frac{1}{f'(f^{-1}(x))}}$$

$$\left(\sqrt[3]{x}\right)' = \frac{1}{3(\sqrt[3]{x})^2} = \frac{1}{3(\sqrt[3]{x^2})}$$

6 Overzicht rekenregels

$$
\begin{array}{ll}
(af+bg)'(x) = af'(x)+bg'(x) \ \ (a,b\in\mathbb{R}) & (\square+\triangle)' = \square'+\triangle' \\
(f\cdot g)'(x) = f'(x)\cdot g(x)+f(x)\cdot g'(x) & (\square\cdot\triangle)' = \square'\cdot\triangle+\square\cdot\triangle' \\
\left(f^r\right)'(x) = r f^{r-1}(x)\cdot f'(x) \ \ (r\in\mathbb{R}) & \left(\square^r\right)' = r\,\square^{r-1}\cdot\square' \\
\left(\frac{1}{f}\right)' = \frac{-f'(x)}{f(x)^2} & \left(\frac{1}{\square}\right)' = \frac{-\square'}{\square^2} \\
\left(\frac{f}{g}\right)'(x) = \frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g(x)^2} & \left(\frac{\square}{\triangle}\right)' = \frac{\square'\cdot\triangle-\square\cdot\triangle'}{\triangle^2} \\
\left(f(g(x))\right)' = f'(g(x))\cdot g'(x) & \\
\left(f^{-1}(x)\right)' = \frac{1}{f'(f^{-1}(x))} &
\end{array}
$$

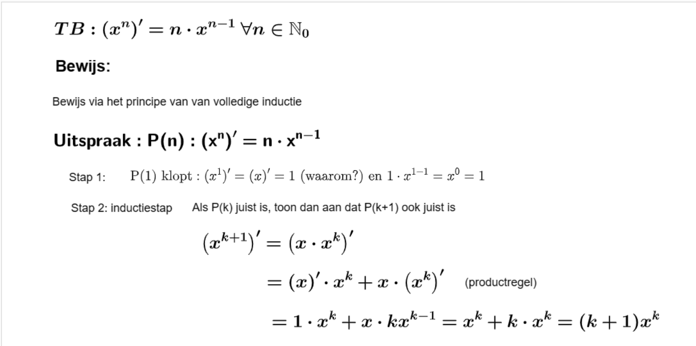
⋮

⋮

$$
\begin{array}{lll}
f(x) = c \ \ (c\in\mathbb{R}) & f'(x) = 0 & \\
f(x) = x & f'(x) = 1 & \\
f(x) = x^r \ (r\in\mathbb{R}) & f'(x) = rx^{r-1} & \left(\square^r\right)' = r\cdot\square^{r-1}\cdot\square' \\
f(x) = \sqrt{x} & f'(x) = \frac{1}{2\sqrt{x}} & \left(\sqrt{\square}\right)' = \frac{1}{2\sqrt{\square}}\cdot\square' \\
f(x) = e^x & f'(x) = e^x & \left(e^{\square}\right)' = e^{\square}\cdot\square' \\
f(x) = a^x \ \ (a\in\mathbb{R}_0^+\backslash\{1\}) & f'(x) = a^x lna & \left(a^{\square}\right)' = a^{\square}lna\cdot\square' \\
f(x) = lnx & \text{f'(x)=1}\frac{}{x} & \left(ln\square\right)' = \frac{1}{\square}\cdot\square' \\
f(x) = \log_a x & f'(x) = \frac{1}{x\ln a} & \left(\log_a\square\right)' = \frac{1}{\square\ln a}\cdot\square' \\
f(x) = \sin x & f'(x) = \cos x & \left(sin\square\right)' = \cos\square\cdot\square' \\
f(x) = \cos x & f'(x) = -\sin x & \left(\cos\square\right)' = -\sin\square\cdot\square' \\
f(x) = \tan x & f'(x) = \frac{1}{\cos^2 x} & \left(\tan\square\right)' = \frac{1}{cos^2\square}\cdot\square' \\
f(x) = \cot x & f'(x) = \frac{-1}{\sin^2 x} & \left(\cot\square\right)' = \frac{-1}{sin^2\square}\cdot\square' \\
f(x) = \arcsin x & f'(x) = \frac{1}{\sqrt{1-x^2}} & \left(\arcsin\square\right)' = \frac{1}{\sqrt{1-\square^2}}\cdot\square' \\
f(x) = \arccos x & f'(x) = \frac{-1}{\sqrt{1-x^2}} & \left(\arccos\square\right)' = \frac{-1}{\sqrt{1-\square^2}}\cdot\square' \\
f(x) = \operatorname{arc\,tan} x & f'(x) = \frac{1}{1+x^2} & \left(\operatorname{arc\,tan}\square\right)' = \frac{1}{1+\square^2}\cdot\square' \\
f(x) = \operatorname{arc\,cot} x & f'(x) = \frac{-1}{1+x^2} & \left(\operatorname{arc\,cot}\square\right)' = \frac{-1}{1+\square^2}\cdot\square'
\end{array}
$$

7 Oefeningen

1. Bewijs afgeleide $\left(x^n\right)' = nx^{n-1}$


$TB : (x^n)' = n \cdot x^{n-1}\ \forall n \in \mathbb{N}_0$

Bewijs:

Bewijs via het principe van van volledige inductie

Uitspraak : P(n) : $(x^n)' = n \cdot x^{n-1}$

Stap 1: P(1) klopt : $(x^1)' = (x)' = 1$ (waarom?) en $1 \cdot x^{1-1} = x^0 = 1$

Stap 2: inductiestap Als P(k) juist is, toon dan aan dat P(k+1) ook juist is

$$\left(x^{k+1}\right)' = \left(x \cdot x^k\right)'$$
$$= (x)' \cdot x^k + x \cdot \left(x^k\right)' \quad \text{(productregel)}$$
$$= 1 \cdot x^k + x \cdot kx^{k-1} = x^k + k \cdot x^k = (k+1)x^k$$

Figure 9: https://www.geogebra.org/m/yvrncth8

2. Op deze site kunnen alle oefeningen gecontroleerd worden: https://www.derivative-calculator.net/

3. afgeleide samengestelde functies: https://www.geogebra.org/m/YN8wHwSh

4. Bereken de afgeleide van onderstaande functies

   (a) $f(t) = (1+t)\ln t$
   (b) $f(t) = t^2 \cos t$
   (c) $f(t) = \dfrac{-1}{t^2}$
   (d) $f(t) = \dfrac{3t-1}{2t+2}$
   (e) $f(x) = \sin(4x+5)$
   (f) $f(t) = \ln(7t^2)$
   (g) $f(x) = \dfrac{4x^3+1}{3x}$
   (h) $f(x) = \tan\dfrac{1}{x}$
   (i) $f(x) = \cos(-8x^2-1)$
   (j) $f(x) = \sin^3(3x)$
   (k) $f(x) = \sqrt{x^2-7x+8}$
   (l) $f(t) = \ln t\, \sin(t^2)$
   (m) $f(t) = \ln(1-t^2)$

5. Bereken de afgeleide van onderstaande functies

   (a) $r(\theta) = 2\,\theta^{1/2} + \dfrac{3}{2}\,\theta^{2/3} + \dfrac{4}{3}\,\theta^{3/4}$
   (b) $r(\theta) = \dfrac{4}{1+2\cos\theta}$
   (c) $r(\theta) = \sqrt{1-2\theta}$
   (d) $r(\theta) = \dfrac{a}{\theta}$ met $a > 0$.

6. Bereken de afgeleide van de volgende functies

   (a) $f(x) = sin(\sqrt[3]{x^2})$
   (b) $f(x) = cos^3(x^3) + (x^3-3)^3$
   (c) $f(x) = \frac{sin^2(4x-1)}{x^2+3x+5}$

7. alle rekenregels door elkaar:

   (a) https://homepages.bluffton.edu/~nesterd/apps/derivs.html
   (b) https://www.math-exercises.com/limits-derivatives-integrals/derivative-of-a-function

8. Bepaal de waarden van $n \in \mathbb{Z}$ zodat $y = x^n$ een oplossing is van volgende (differentiaal)vergelijking: $x^2y" - 2xy' = 4y$ (A: n=4 en n=-1)

9. Bepaal m.b.v. $f(x) = (2x+1)^{\frac{3}{2}}$ een lineaire benadering voor $83^{\frac{3}{2}}$. (A. 756)

10. Gegeven de functie $f : \mathbb{R} \mapsto \mathbb{R} : x \mapsto f(x) = x^2 - x^3$ en de functie $g : \mathbb{R} \mapsto \mathbb{R} : x \mapsto g(x) = f(2x-1)$. Bepaal de afgeleide $g'(0)$. (A. -10)

11. Bereken de gevraagde afgeleide m.b.v. onderstaande tabel

Use the following table to answer question questions # 9 and # 10.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	1	3	2
2	2	2	1	3
3	3	1	3	1

9. If $h(x) = f(g(x))$, what is $h'(1)$?

10. If $H(x) = g(f(x))$, what is $H'(3)$?

12. Bereken de gevraagde afgeleide m.b.v. onderstaande tabel

t	0	1	2	3	4
$h(t)$	-2	2	3	4	8
$h'(t)$	3.5	0.5	2.5	1.5	5
$h''(t)$	6	0.25	0.3	-0.4	0.6

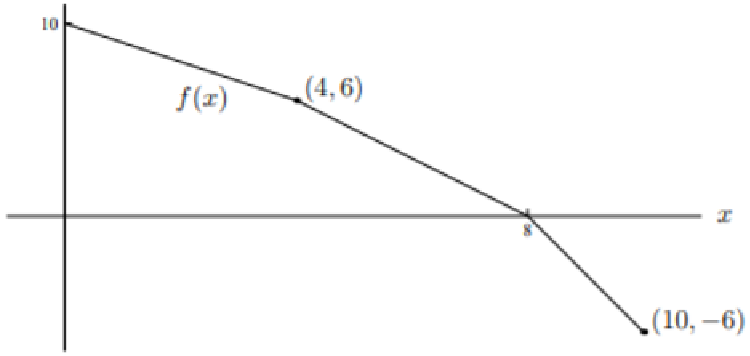
(a) $a(t) = h(t^2 - 1)$ gevraagd $a'(3)$

(b) $b(t) = \frac{h(t)}{t^2}$ gevraagd $b'(4)$

(c) $c(t) = h^{-1}(t)$ gevraagd $c'(2)$

13. Bepaal g'(3) als

Consider the piecewise linear function $f(x)$ graphed below:



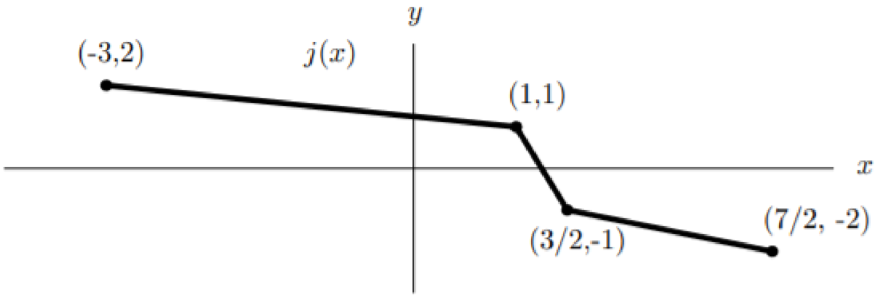
(a) $g(x) = \frac{f(x^2)}{x}$ (antw. $-\frac{51}{9}$)

(b) $g(x) = \sin\left(f(x)^3\right)$ (antw. $-147cos(343)$)

(c) $g(x) = f^{-1}(x)$ (antw. $\frac{-2}{3}$)

14. Bereken de gevraagde afgeleiden

[15 points] The graph of a piecewise linear function $j(x)$ is given below. Use it to select the correct value of each derivative below. Circle only one answer for each part. Ambiguous marks will receive no credit.



a. [3 points] $\frac{d}{dx}[j(4\cos x)]$ at $x = \frac{\pi}{4}$.

(A) $-1/2$
(B) $\sqrt{2}$
(C) $-\sqrt{2}/2$
(D) $-\sqrt{2}$

b. [3 points] $\frac{d}{dx}[j(j(x))]$ at $x = 2$.

(A) $1/4$
(B) $1/8$
(C) $-1/4$
(D) $-1/8$

15. Toepassing:

If x is the amount of antibiotic taken orally (in mg), then the function h(x) gives the amount entering the bloodstream through the stomach. If x mg reaches the blood-stream, then g(x) gives the amount that survives filtration by the liver. Finally, if x mg survives filtration by the liver, then f(x) is absorbed into the sinus cavity. Thus, for a given dose x, the amount making it to the sinus cavity is $A(x) = f(g(h(x)))$. Suppose that a dose of 500 mg is given, $h(500) = 8$, $g(8) = 2$, $f(2) = 1.5$, and $h'(500) = 2.5$, $g'(8) = \frac{1}{4}$, and $f'(2) = 1$. Calculate $A'(x)$ and interpret your answer.

MATH 171 - Derivative Worksheet

Differentiate these for fun, or practice, whichever you need. The given answers are not simplified.

1. $f(x) = 4x^5 - 5x^4$
2. $f(x) = e^x \sin x$
3. $f(x) = (x^4 + 3x)^{-1}$
4. $f(x) = 3x^2(x^3 + 1)^7$
5. $f(x) = \cos^4 x - 2x^2$
6. $f(x) = \dfrac{x}{1 + x^2}$
7. $f(x) = \dfrac{x^2 - 1}{x}$
8. $f(x) = (3x^2)(x^{\frac{1}{2}})$
9. $f(x) = \ln(xe^{7x})$
10. $f(x) = \dfrac{2x^4 + 3x^2 - 1}{x^2}$
11. $f(x) = (x^3)\sqrt[5]{2 - x}$
12. $f(x) = 2x - \dfrac{4}{\sqrt{x}}$
13. $f(x) = \dfrac{4(3x - 1)^2}{x^2 + 7^x}$
14. $f(x) = \sqrt{x^2 + 8}$
15. $f(x) = \dfrac{x}{\sqrt{1 - (\ln x)^2}}$
16. $f(x) = \dfrac{6}{(3x^2 - \pi)^4}$
17. $f(x) = \dfrac{(3x^2 - \pi x)^4}{6}$
18. $f(x) = \dfrac{x}{(x^2 + \sqrt{3x})^5}$
19. $f(x) = (xe^x)^\pi$
20. $f(x) = \Big[\arctan(2x)\Big]^{10}$
21. $f(x) = (e^{2x} + e)^{\frac{1}{2}}$
22. $f(x) = (x^6 + 1)^5(4x + 7)^3$
23. $f(x) = (7x + \sqrt{x^2 + 3})^6$
24. $f(x) = \dfrac{\frac{1}{x} + \frac{1}{x^2}}{x - 1}$
25. $f(x) = \sqrt[3]{x^2} - \dfrac{1}{\sqrt{x^3}}$
26. $f(x) = \sqrt{\dfrac{2x + 5}{7x - 9}}$
27. $f(x) = \dfrac{\sin x}{\cos x}$
28. $f(x) = e^x(x^2 + 3)(x^3 + 4)$
29. $f(x) = \dfrac{5x^2 - 7x}{x^2 + 2}$
30. $f(x) = \Big[\ln(5x^2 + 9)\Big]^3$
31. $f(x) = \ln(5x^2 + 9)^3$
32. $f(x) = \cot(6x)$
33. $f(x) = \sec^2 x \cdot \tan x$
34. $f(x) = \arcsin(2^x)$
35. $f(x) = \tan(\cos x)$
36. $f(x) = [(x^2 - 1)^5 - x]^3$
37. $f(x) = \sec x \cdot \sin(3x)$
38. $f(x) = \dfrac{(x - 1)^3}{x(x + 3)^4}$
39. $f(x) = \log_5(3x^2 + 4x)$

In problems 40 – 42, find $\dfrac{dy}{dx}$. Assume y is a differentiable function of x.

40. $3y = xe^{5y}$
41. $xy + y^2 + x^3 = 7$
42. $\dfrac{\sin y}{y^2 + 1} = 3x$

If f and g are differentiable functions such that $f(2) = 3$, $f'(2) = -1$, $f'(3) = 7$, $g(2) = -5$ and $g'(2) = 2$, find the numbers indicated in problems 43 – 48.

43. $(g - f)'(2)$
44. $(fg)'(2)$
45. $\left(\dfrac{f}{g}\right)'(2)$
46. $(5f + 3g)'(2)$
47. $(f \circ f)'(2)$
48. $\left(\dfrac{f}{f + g}\right)'(2)$

Answers: Absolutely not simplified ... you should simplify more.

1. $f'(x) = 20x^4 - 20x^3$

2. $f'(x) = e^x \cos x + (\sin x)e^x$

3. $f'(x) = -1(x^4 + 3x)^{-2}(4x^3 + 3)$

4. $f'(x) = 3x^2 \cdot 7(x^3 + 1)^6(3x^2) + (x^3 + 1)^7 \cdot 6x$

5. $f'(x) = 4(\cos x)^3(-\sin x) - 4x$

6. $f'(x) = \dfrac{(1 + x^2)(1) - x(2x)}{(1 + x^2)^2}$

7. $f'(x) = 1 + x^{-2}$ (*Simplify f first.*)

8. $f'(x) = 3 \cdot \dfrac{5}{2}\, x^{\frac{3}{2}}$ (*Simplify f first.*)

9. $f'(x) = \dfrac{1}{x} + 7$ (*Simplify f first.*)

10. $f'(x) = 4x + 0 + 2x^{-3}$ (*Simplify f first.*)

11. $f'(x) = x^3 \cdot \dfrac{1}{5}(2 - x)^{\frac{-4}{5}}(-1) + (2 - x)^{\frac{1}{5}}(3x^2)$

12. $f'(x) = 2 + 2x^{\frac{-3}{2}}$

13. $f'(x) = \dfrac{(x^2 + 7^x)\Big[4 \cdot 2(3x - 1)(3)\Big] - 4(3x - 1)^2(2x + 7^x \ln 7)}{(x^2 + 7^x)^2}$

14. $f'(x) = \dfrac{1}{2}(x^2 + 8)^{\frac{-1}{2}}(2x)$

15. $f'(x) = \dfrac{\Big(1 - (\ln x)^2\Big)^{\frac{1}{2}}(1) - x \cdot \frac{1}{2}\Big(1 - (\ln x)^2\Big)^{\frac{-1}{2}}\Big(-2(\ln x) \cdot \frac{1}{x}\Big)}{1 - (\ln x)^2}$

16. $f'(x) = -24(3x^2 - \pi)^{-5}(6x)$

17. $f'(x) = \dfrac{1}{6}\Big[4(3x^2 - \pi x)^3(6x - \pi)\Big]$

18. $f'(x) = \dfrac{(x^2 + \sqrt{3x})^5(1) - x\Big[5(x^2 + \sqrt{3x})^4\Big(2x + \frac{1}{2}(3x)^{\frac{-1}{2}} \cdot 3\Big)\Big]}{(x^2 + \sqrt{3x})^{10}}$

19. $f'(x) = \pi(xe^x)^{(\pi-1)}\Big[xe^x + e^x\Big]$

20. $f'(x) = 10\Big[\arctan(2x)\Big]^9 \cdot \dfrac{1}{1 + (2x)^2} \cdot 2$

21. $f'(x) = \dfrac{1}{2}(e^{2x} + e)^{\frac{-1}{2}}(e^{2x} \cdot 2 + 0)$

22. $f'(x) = (x^6 + 1)^5\Big[3(4x + 7)^2(4)\Big] + (4x + 7)^3\Big[5(x^6 + 1)^4(6x^5)\Big]$

23. $f'(x) = 6(7x + \sqrt{x^2 + 3})^5\Big(7 + \dfrac{1}{2}(x^2 + 3)^{\frac{-1}{2}} \cdot 2x\Big)$

24. $f'(x) = \dfrac{(x - 1)(-x^{-2} - 2x^{-3}) - (x^{-1} + x^{-2})(1)}{(x - 1)^2}$

25. $f'(x) = \dfrac{2}{3}x^{\frac{-1}{3}} + \dfrac{3}{2}x^{\frac{-5}{2}}$

26. $f'(x) = \dfrac{1}{2}\left(\dfrac{2x + 5}{7x - 9}\right)^{\frac{-1}{2}}\left[\dfrac{(7x - 9)(2) - (2x + 5)(7)}{(7x - 9)^2}\right]$

27. $f'(x) = \sec^2 x$

28. $f'(x) = \Big[e^x(x^2 + 3)\Big](3x^2) + (x^3 + 4)\Big[e^x(2x) + (x^2 + 3)e^x\Big]$

29. $f'(x) = \dfrac{(x^2 + 2)(10x - 7) - (5x^2 - 7x)(2x)}{(x^2 + 2)^2}$

30. $f'(x) = 3\Big[\ln(5x^2 + 9)\Big]^2 \cdot \dfrac{1}{5x^2 + 9}(10x + 0)$

31. $f'(x) = \dfrac{1}{(5x^2 + 9)^3} \cdot \Big[3(5x^2 + 9)^2(10x + 0)\Big]$

32. $f'(x) = -\csc^2(6x) \cdot 6$

33. $f'(x) = \sec^2 x(\sec^2 x) + \tan x\Big[2 \cdot \sec x(\sec x \tan x)\Big]$

34. $f'(x) = \dfrac{1}{\sqrt{1 - (2^x)^2}} \cdot 2^x \ln 2$

35. $f'(x) = \Big(\sec^2(\cos x)\Big)(-\sin x)$

36. $f'(x) = 3\Big[(x^2 - 1)^5 - x\Big]^2\Big(5(x^2 - 1)^4 \cdot 2x - 1\Big)$

37. $f'(x) = \sec x\Big(\cos(3x) \cdot 3\Big) + \sin(3x)\Big(\sec x \tan x\Big)$

38. $f'(x) = \dfrac{x(x + 3)^4\Big[3(x - 1)^2(1)\Big] - (x - 1)^3\Big[x \cdot 4(x + 3)^3(1) + (x + 3)^4(1)\Big]}{x^2(x + 3)^8}$

39. $f'(x) = \dfrac{1}{(3x^2 + 4x) \cdot \ln 5} \cdot (6x + 4)$

40. $\dfrac{dy}{dx} = \dfrac{e^{5y}}{3 - 5xe^{5y}}$

41. $\dfrac{dy}{dx} = \dfrac{-3x^2 - y}{x + 2y}$

42. $\dfrac{dy}{dx} = \dfrac{3(y^2 + 1)^2}{(y^2 + 1)(\cos y) - 2y \sin y}$

43. 3

44. 11

45. $\dfrac{-1}{25}$

46. 1

47. -7

48. $\dfrac{-1}{4}$

Practice Problems on Derivative Computing (with Solutions)

This problem set is generated by Di. All of the problems came from past exams of Math 221. For derivative computing – unlike many of other math concepts – more lectures do not help much, and nothing compares to practicing on one's own! The idea of this problem set is to get enough practice, till the point that it becomes hard to make any mistake. :)

1. $y = \sqrt[3]{x^2}\sin x$
2. $y = \frac{\tan x}{x^3+2}$
3. $y = \left(x^4 + \sin x \cos x\right)^3$
4. $y = \frac{x^3-2x}{x+3}$
5. $y = \left(\sqrt{x^2-1}+1\right)^{10}$
6. $y = \cos\left(x^2\right)\tan\left(\sqrt{x+1}\right)$
7. $y = \cos\left(\cos\left(\cos\left(3x\right)\right)\right)$
8. $y = \sqrt{\frac{1+x}{2-x}}$
9. $y = x^2\left(\sqrt{x}+2\right)$
10. $f\left(x\right) = 2\sqrt{x^2+1} + \sin\left(\frac{4\pi}{5}\right)$
11. $h\left(x\right) = \frac{\sin x}{2x-3}$
12. $y = x^3 + \frac{1}{2x^3} - \frac{2}{\sqrt[3]{x}}$
13. $y = \sin\left(5x\right)\cos\left(3x\right)$
14. $y = \left(\cos\left(x^2\right) + \cos^2 x\right)^4$
15. $y = 12 + x\cos x + x^5$
16. $y = \left(x^2+x-1\right)\sin x \cos^2 x$
17. $y = \cos\left(x^2 + \frac{x}{x+1}\right)$
18. $y = \frac{x^2-2}{x^4+1}$
19. $y = x^2\sqrt[3]{\tan x}$
20. $y = \left(\sqrt{x^2+1}+x\right)^5$
21. $y = \sqrt{1-\cos x}\left(\tan x\right)^3$
22. $y = \frac{\sin\left(x^3\right)}{\sin\left(x^2\right)}$
23. $y = x^3 + \sin\left(x\right)\cos^2\left(x\right)$

24. $y = x(10x+6)^{2011}$

25. $y = \sqrt{(\sin x)^3 + 1}$

26. $y = \tan\left(x^4 + 3x^2 + 1\right)$

27. $y = \frac{\sin(3x)}{1+x^4}$

28. $y = (5 - 2\cos x)^{\frac{3}{2}}$

29. $y = (5\sqrt{x} + 3)^{80}$

30. $y = \frac{1}{x}\sin^{-4}(x) - \frac{x}{3}\cos^3(x)$

31. $y = \frac{\tan(2x)}{(x+5)^4}$

32. $y = \tan\left(\frac{\cos(x)}{x}\right)$

33. $y = \sin\left(\frac{x}{\sqrt{x^2+1}}\right)$

34. $y = \sin^5\left(3x^4 - 7x\right)$

The following problems involve e^x and $\ln x$, which we haven't seen in our lecture so far, but we will learn them later. We could practice them later when the material is covered. To do them, you need to know:

$$(e^x)' = e^x,\ (\ln x)' = \frac{1}{x},\ \frac{d\arctan x}{dx} = \frac{1}{1+x^2}.$$

1. $f(x) = x\ln\left(e^{2x} + 2\right)$
2. $y = \arctan\left(e^{4x} + 3x\right)$
3. $y = \ln\left(x + \sqrt{x^2 - 1}\right)$
4. $y = 3\ln(x\sin x)$
5. $y = e^{-\tan(x+1)}$
6. $y = \sin\left(\ln x + 3x^2\right)$
7. $y = \frac{e^{x^2}}{x+3}$
8. $y = \sqrt{\ln x + 1}$
9. $y = e^{\sqrt{x^2+1}}$
10. $y = \ln\left(\frac{2\left(1+x^2\right)}{x^4}\right)$
11. $y = \ln e^{2x}$
12. $y = \arctan(x-1) + \sqrt{\sin(\ln x)}$
13. $y = x^2 e^{3x^2 - 5x}$
14. $y = \ln(4x+6)\, e^{5x}$

Solutions:

1. $y = x^{\frac{3}{2}} \sin x$

   $y' = \frac{3}{2} x^{\frac{1}{2}} \sin x + x^{\frac{3}{2}} \cos x$

2. $y' = \frac{\sec^2 x\left(x^3+2\right) - 3x^2 \tan x}{\left(x^3+2\right)^2}$

3. $y' = 3\left(x^4 + \sin x \cos x\right)^2 \left(4x^3 + \cos^2 x - \sin^2 x\right)$

4. $y' = \frac{\left(3x^2-2\right)(x+3) - \left(x^3-2x\right)}{(x+3)^2}$

5. $y = \left(\left(x^2-1\right)^{\frac{1}{2}} + 1\right)^{10}$

   $y' = 10\left(\sqrt{x^2-1} + 1\right)^9 \frac{1}{2}\left(x^2-1\right)^{-\frac{1}{2}} 2x$

6. $y' = -\sin\left(x^2\right) 2x \tan\left(\sqrt{x+1}\right) + \cos\left(x^2\right) \sec^2\left(\sqrt{x+1}\right) \frac{1}{2}(x+1)^{-\frac{1}{2}}$

7. $y' = -\sin(\cos(\cos(3x))) \cdot (-\sin(\cos(3x))) \cdot (-\sin(3x)) \cdot 3$

8. $y = \left(\frac{1+x}{2-x}\right)^{\frac{1}{2}}$

   $y' = \frac{1}{2}\left(\frac{1+x}{2-x}\right)^{-\frac{1}{2}} \frac{(2-x)+(1+x)}{(2-x)^2} = \frac{1}{2}\left(\frac{1+x}{2-x}\right)^{-\frac{1}{2}} \frac{3}{(2-x)^2}$

9. $y = x^2\left(x^{\frac{1}{2}} + 2\right)$

   $y' = 2x\left(x^{\frac{1}{2}} + 2\right) + x^2\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$

10. $f(x) = 2\left(x^2+1\right)^{\frac{1}{2}} + \sin\left(\frac{4\pi}{5}\right)$ tricky question! notice $\sin\left(\frac{4\pi}{5}\right)$ is just a constant.

    $f'(x) = \left(x^2+1\right)^{-\frac{1}{2}} 2x$

11. $h'(x) = \frac{\cos x(2x-3) - 2\sin x}{(2x-3)^2}$

12. $y = x^3 + \frac{1}{2}x^{-3} - 2x^{-\frac{1}{3}}$

    $y' = 3x^2 - \frac{3}{2}x^{-4} + \frac{2}{3}x^{-\frac{4}{3}}$

13. $y' = 5\cos(5x)\cos(3x) + \sin(5x)(-\sin(3x))\,3$

14. $y' = 4\left(\cos\left(x^2\right) + \cos^2 x\right)^3 \left(-\sin\left(x^2\right) 2x + 2\cos x(-\sin x)\right)$

15. $y' = \cos x + x(-\sin x) + 5x^4$

16. This problem is the product rule of 3 functions. Just kill each of them at one time. $(abc)' = a'bc + ab'c + abc'$.

    $y' = (2x+1)\sin x \cos^2 x + \left(x^2+x-1\right)\cos^3 x + \left(x^2+x-1\right)\sin x 2\cos x(-\sin x)$

17. $y' = -\sin\left(x^2 + \frac{x}{x+1}\right)\left(2x + \frac{x+1-x}{(x+1)^2}\right) = -\sin\left(x^2 + \frac{x}{x+1}\right)\left(2x + \frac{1}{(x+1)^2}\right)$

18. $y' = \frac{2x\left(x^4+1\right) - 4x^3\left(x^2-2\right)}{\left(x^4+1\right)^2}$

19. $y = x^2 (\tan x)^{\frac{1}{3}}$
$y' = 2x (\tan x)^{\frac{1}{3}} + x^2 \frac{1}{3} (\tan x)^{-\frac{2}{3}} \sec^2 x$

20. $y' = 5 \left(\sqrt{x^2+1} + x\right)^4 \left(\frac{1}{2} \left(x^2+1\right)^{-\frac{1}{2}} 2x + 1\right)$

21. $y = (1 - \cos x)^{\frac{1}{2}} (\tan x)^3$
$y' = \frac{1}{2} (1 - \cos x)^{-\frac{1}{2}} \sin x (\tan x)^3 + (1 - \cos x)^{\frac{1}{2}} 3 (\tan x)^2 \sec^2 x$

22. $y' = \frac{\cos\left(x^3\right) 3x^2 \sin\left(x^2\right) - \cos\left(x^2\right) 2x \sin\left(x^3\right)}{\left[\sin\left(x^2\right)\right]^2}$

23. $y' = 3x^2 + \cos^3 x + \sin x \cdot 2 \cos x (-\sin x)$

24. $y' = (10x+6)^{2011} + x2011 (10x+6)^{2010} 10$

25. $y = \left((\sin x)^3 + 1\right)^{\frac{1}{2}}$
$y' = \frac{1}{2} \left((\sin x)^3 + 1\right)^{-\frac{1}{2}} 3 (\sin x)^2 \cos x$

26. $y' = \sec^2 \left(x^4 + 3x^2 + 1\right) \left(4x^3 + 6x\right)$

27. $y' = \frac{\cos(3x) 3 \left(1+x^4\right) - 4x^3 \sin(3x)}{\left(1+x^4\right)^2}$

28. $y' = \frac{3}{2} (5 - 2\cos x)^{\frac{1}{2}} (2 \sin x)$

29. $y' = 80 (5\sqrt{x} + 3)^{79} \left(\frac{5}{2} x^{-\frac{1}{2}}\right)$

30. For the 1st term, you can also do quotient rule. Here I rewrite it as a product, so that I can kill people. :)
$y = x^{-1} \sin^{-4}(x) - \frac{x}{3} \cos^3 x$
$y' = -x^{-2} \sin^{-4}(x) + x^{-1} (-4) \sin^{-5}(x) \cos(x) - \frac{1}{3} \cos^3 x - \frac{x}{3} 3 \cos^2 x (-\sin x)$

31. $y' = \frac{\sec^2(2x) 2 (x+5)^4 - 4 (x+5)^3 \tan(2x)}{(x+5)^8}$

32. $y' = \sec^2 \left(\frac{\cos x}{x}\right) \frac{-\sin x \cdot x - \cos x}{x^2}$

33. $y' = \cos \left(\frac{x}{\sqrt{x^2+1}}\right) \frac{\sqrt{x^2+1} - \frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}} 2x^2}{x^2+1}$

34. $y' = 5 \sin^4 \left(3x^4 - 7x\right) \left(12x^3 - 7\right)$

Well, I guess no one works till the last problem together with me... I admit computing derivatives could be pretty boring and exhausted. But if you finished all of these problems, I believe derivative computing will become part of your nature, and you can do them quickly and accurately. Now it's time to say: I came, I calculated, I conquered. :)

Some irrelevant aside by Di: *Typing solutions is also very exhausted! Half way through typing, I started to question myself why do I want to tortune myself on doing this extra amount of work.*