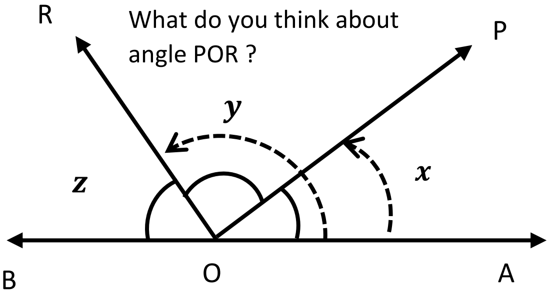
Sin(A$\pm$B) and Cos(A$\pm$B) Identities

Objective: To investigate compound angle formula or identity for sine and cosine functions	**Pre-Required Knowledge:** Basics of trigonometry, Sine rule and cosine rule.

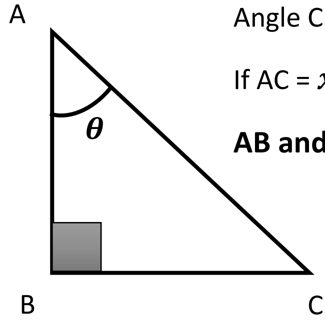
Starter of the Day

What do you think about angle POR ?



Angle C =

If AC = x **units, determine AB and BC**

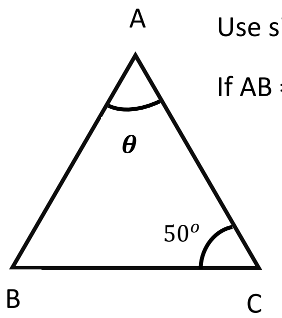


$$sine\ rule: \frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

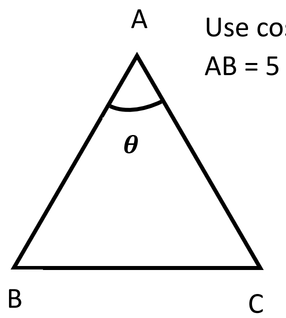
$$cosine\ rule: \cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

Use sine rule to figure out $\boldsymbol{\theta}$

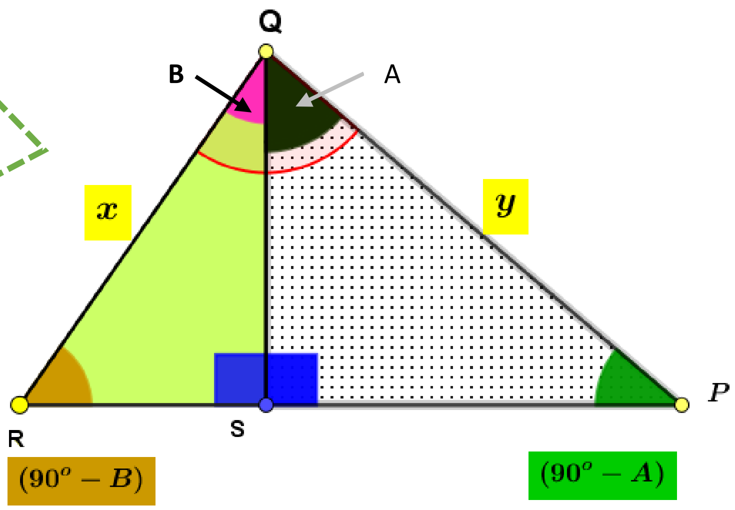
If AB = 5 cm, BC= 4 cm



Use cosine rule to figure out $\boldsymbol{\theta}$ If AB = 5 cm, BC= 4 cm & AC=6 cm



There are two right-angled triangles named as **PSQ and RSQ** with marked angles.
Angle **PQS** is denoted by **A** (black shaded angle) and Angle **SPQ = 90-A**.
Angle **RQS** is denoted by **B** (turquoise shaded angle) & Angle **SRQ= 90-B**.



Task 1 If P and R are on the opposite side of S then Angle PQR in terms of A and B

Angle PQR=

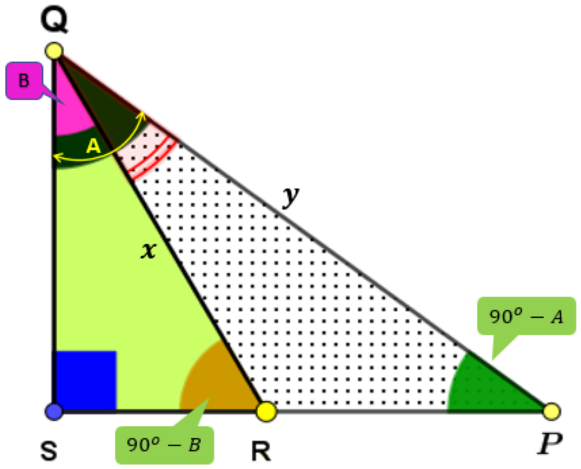
Use basics of trigonometry in ΔPQS & figure out sides PS and QS if PQ $= y$ units. Give your answers in terms of y and $angle\ A$.

PS= **QS=**

If the points P and R are on the right hand side of S then determine Angle PQR in terms of A and B

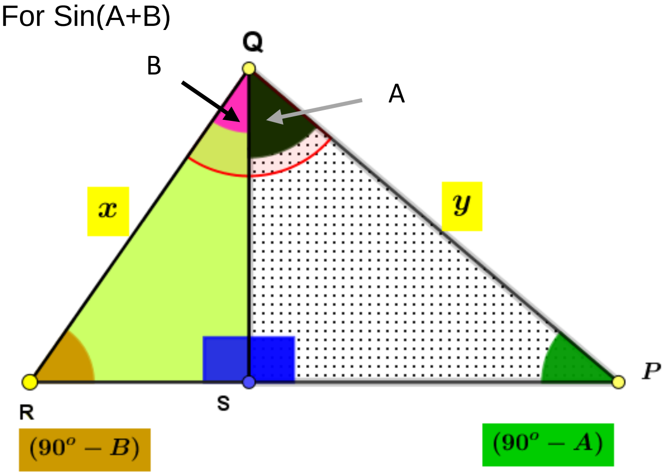
Angle PQR=

Work out the sides RS and QS of triangle RQS if $RQ = x$ units.
(give your answers in terms of x and $angle\ B$)



RS= **QS=**

For Sin(A+B)



Use all results:

$if\ Angle\ PQR = A + B, then$

$PR =$

$QS =$ $=$

$\Rightarrow \quad \frac{x}{y} =$

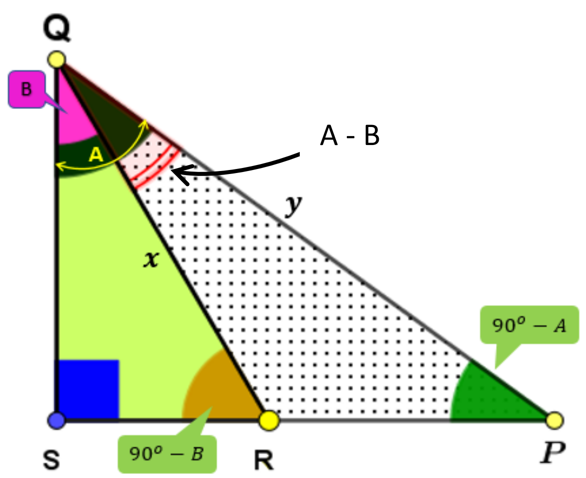
$Apply\ sine\ rule: \quad \frac{\sin(PQR)}{PR} = \frac{\sin(90^o - A)}{RQ}$

Using above values, simplify the expression for $\sin(A+B)$ only in terms of sine or cosine with angles A and B.

Hint: you should get rid of x *and* y using $\frac{x}{y} = \frac{\cos(A)}{\cos(B)}$ $(using\ two\ values\ of\ QS)$

$\sin(A+B) =$

For Sin (A-B)



Use all results:

$if\ Angle\ PQR = A - B, then$

$PR =$

$QS = \qquad\qquad =$

$\Rightarrow \quad \frac{x}{y} =$

Now apply $sine\ rule$:

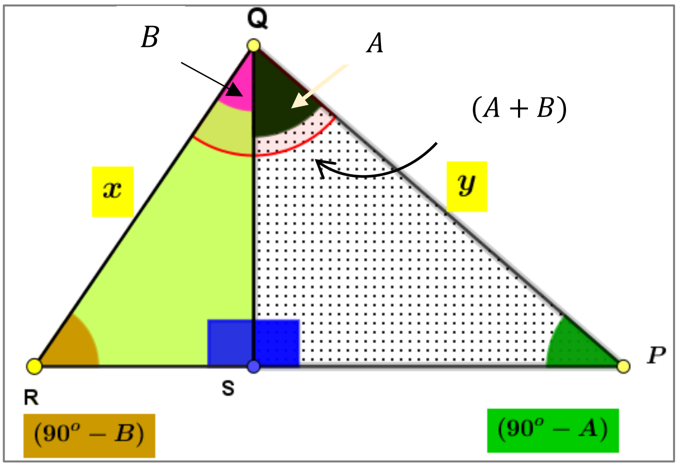
$$\frac{\sin(A-B)}{PR} = \frac{\sin(90^o - A)}{x}$$

Simpify and write an expresión for sin(A-B)

$\sin(A-B) =$

Part 2: Cosine Identities

$\cos(A+B)$



Using all the values you have used for

sin (A + B). let´s apply cosine rule

since PR is opposite of angle(PQR) &

x, y *are adjacent sides.*

so, $\cos(PQR) = \frac{x^2 + y^2 - (PR)^2}{2xy}$

Now simplify

You may need to use the following:

1. $1 - \sin^2\theta = \cos^2\theta$
2. $\frac{x}{y} = \frac{\cos(A)}{\cos(B)}$ *or* $\frac{y}{x} = \frac{\cos(B)}{\cos(A)}$

$\cos(A+B) =$

What do you think about Cos(A- B)?

$\cos(A-B) =$