Why Learn LCM, GCF, and Prime Numbers?

In elementary school you were introduced **prime numbers**, you learned them and probably forgot them. For many students, the lack of lessons and problems using them let them think why you would need to know them. Middle school math teachers may discuss them, but they seldom used them to solve problems. Finally, curriculum designers sometimes leave out important ideas assuming teachers will teach it. (See pages 9-12 review Prime Numbers.)

The reason you should learn about prime numbers is their uniqueness as a number. A **prime number** is a value which has two ***unique*** factors, the number itself and one (1). More importantly, student knowledge of prime numbers can give them insight into the factors of other number types: composite numbers, fractional numbers, etc. **Composite numbers** have three or more factors. Composites can have many different factor sets, but they have only one prime factor set. This fact can be used to assist in division, solving problems involving fractions as in finding common denominators, common factors, equivalent fractions (which are needed in fraction addition and subtraction), and many applications in algebra and beyond.

Examples:

In elementary school, students find factor pairs (2 numbers that multiply to give the product) of 12, 15, 18, and 24.

Factor pairs for 12: {1, 12}, {2, 6}, {3, 4}, {4, 3}, {6, 2}, {12, 1}

You will notice that some of the pairs are just the swapping the values, so we do not include both?

Factor pairs for 15: {1, 15}, {3, 5}

Factor pairs for 18: {1, 18}, {2, 9}, {3, 6}

Factor pairs for 24: {1, 24}, {2, 12}, {3, 8}, {4, 6}

Factor pairs for 36: {1, 36}, {2, 18}, {3, 12}, {4, 9}, {6, 6} **Square numbers** repeat numbers in one pair.

Factor pairs of 144 are:

{1, 144},{2, 72},{3, 48},{4, 38},{6, 24},{8, 18},{9, 16)},{12, 12} **Advanced**

To simplify Factor Pairs, we can write the Factor Set of the factors of the number.

The factor set of 12: {1, 2, 3, 4, 6, 12}

The factor set of 15: {1, 3, 5, 15}

The factor set of 18: {1, 2, 3, 6, 9, 18}

The factor set of 24: {1, 2, 3, 4, 6, 8, 12, 24}

The factor set of 36: {1, 2, 3, 4, 6, 9, 12, 18, 36} **Square numbers** have an odd number of factors.

The factor set of 144 is: {1, 4, 2, 3, 6, 8, 16, 9, 12, 18, 24, 36, 48, 72, 144}

Prime factorizations are even a further simplification of the factor set.

The prime factors of 12: 2 x 2 x 3 or 2^2 x 3

The prime factors of 15: 3 x 5

The prime factors of 18: 2 x 3 x 3 or 2 x 3^2

The prime factors of 24: 2 x 2 x 2 x 3 or 2^3 x 3

The prime factors of 36: 2 x 2 x 3 x 3 or 2^2 x 3^2

The Prime factors of 144: $2 \times 2 \times 2 \times 2 \times 3 \times 3$ or $2^4 \times 3^2$.

There is **only** one set of prime numbers for any number.

If we examine different factor pairs from any example above, those pairs will have a prime factor set for the pair. I.e., {12, 12} is also $\{\{2 \times 2 \times 3\}, \{2 \times 2 \times 3\}\}$.

Comparing prime factorizations of different number we can find the Lowest Common Multiple, LCM, or (Lowest Common Denominator, LCD) of two or more numbers or we can find the Greatest Common Factor of those numbers. Tools use for fraction operations and many other mathematical activities.

Find the LCM and GCF of 12 and 9

Multiples:

12, 24, 36, 48, 60, 72,…

9, 18, 27, 36, 45, 54, 63, 72,…

The LCM(12, 9) is 36.

Factor sets:

12: {1, 2, 3, 4, 6, 8, 12}

9: {1, 3, 9}

The GCF(12, 9) is 3.

A prime factorization can find both the LCM and GCF!

12: 2 x 2 x 3

9: 3 x 3

LCM: 2 x 2 x 3 x 3 = 36

Find the LCM and GCF of 14 and 16

Multiples:

14, 28, 42, 56, 70, 84, 98, 112, 126, 140, 154,…

16, 32, 48, 64, 80, 96, 112, 128, 142,…

Factor list:

14: {1, 2, 7, 14}

16: {1, 2, 4, 8, 16}

Prime factorizations:

14: 7 x 2

16: 2 x 2 x 2 x 2

LCM: 7 x 2 x 2 x 2 x 2 = 112

Some Basic Math Definitions

Addition:

Addend1 + **Addend2** = **Sum**
Sum – **Addend1** = **Addend2**
Sum – Addend2 = Addend1

Every addition can be rewritten as two subtraction forms.

Subtraction:

Minuend – **Subtrahend** = **Difference**

Minuend: The number that is to be subtracted from.
Subtrahend: The number that is to be subtracted.
Difference: The result of subtracting one number from another.

Multiplication:

… (in its simplest form) **repeated addition**
Here we see that 6 + 6 + 6 (three 6s) make 18

$$6 \times 3 = 18$$

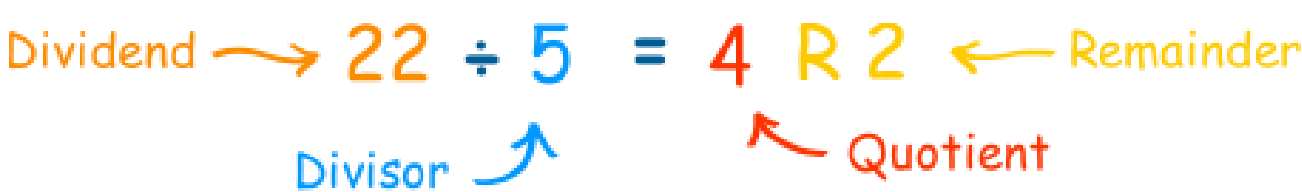
Factor × **Factor** = **18**
Multiplier × **Multiplicand** = **Product**

It can also be said that 3+3+3+3+3+3 (six 3s) make 18

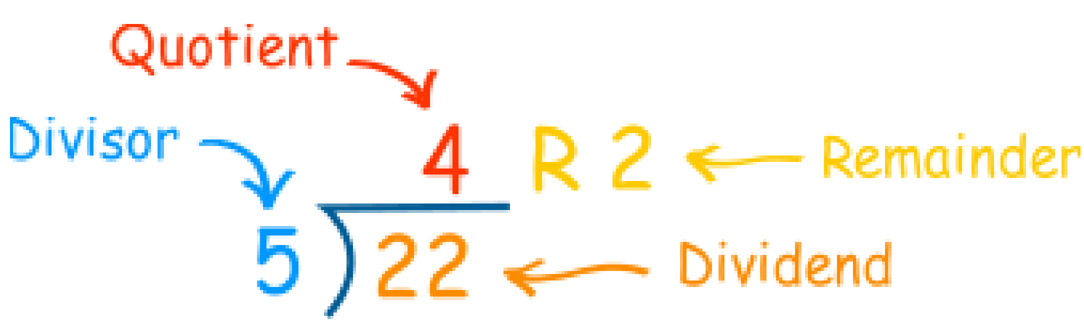
Division:

... splitting into equal parts or groups. It is the result of "fair sharing".

Division has its own special words to remember.
Let's take the simple question of **22 divided by 5**. The answer is **4**, with **2** left over.
Here we see the important words:



Which can also be in this form:



If the Numerator is bigger than the Denominator, you must change answer to a Mixed Number.

The Numerator is a.k.a. the Dividend (it inside the divide symbol), the Remainder is the Numerator of the reduced fraction.
The Divisor is a.k.a. the Denominator.

_Quotient
Divisor) Dividend, what was the Numerator

____
Remainder, becomes the new Numerator.

Lowest Common Multiple (LCM)

Station WTAW is having a promotion in which every 12th caller receives two free concert tickets and every 15th caller receives a limo rental for a night out on the town. The station only has 12 sets of concert tickets and 8 limo rides. Will any caller win be able to win both? If yes, which caller(s) would get both tickets and a limo ride?

To solve this one needs to find out if there is any value(s) where the 12th and the 15th call could occur at the same time. Joey wanted to take Maria to this concert in a limo. He decided to find out which callers would get the tickets or the limo rides.

12: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, ... get tickets

15: 15, 30, 45, 60, 75, 90, 105, 120, 135, 150, ... get limo rides

Joey found out the 60th and 120th callers would get both tickets and a ride. Good luck Joey.

Saul heard the same broadcast and found the same solution but he used another method to find the Lowest Common Multiple. He had just learned about using prime factorization. This is his work:

12: 2 x 2 x 3

15: 3 x 5

LCM: 2 x 2 x 3 x 5 = 60

Saul used the LCM to decide that the 60th and 120th callers would get both tickets and a ride. Saul hopes to invite Juanita to the concert. Good luck Saul.

Both methods are correct solutions for the same problem, you can choose either method for solving problems. However, these examples are only one reason to learn about the LCM and the GCF. The LCM is also known as the Lowest Common Denominator (LCD) when working with fraction addition and subtraction. The GCF helps to reduce fractions to lowest terms. Used together they make adding and subtracting fractions easier.

The times table is essentially a table with a list of the **multiples** of column or row numbers:

The multiples of 2 are the even numbers:
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, …
The multiples of 3 are the following numbers:
3, 6, 9, 12, 15, 18, 21, 24, 27, …
The multiples of 5 are the following numbers:
5, 10, 15, 20, 25, 30, 35, 40, 45, …

These three sets of multiples are those for the three smallest **prime numbers** which are the values students are expected know the **divisibility tests**. Divisibility by 2 is all even numbers, divisibility by 3 is the sum of the digits divide by 3, and divisibility by 5 is any numbers which end in 0 or 5. (02 Math Reference Pages)

If we look at the list of the multiples of 2 and 3 and circle the **common multiples**, we have.

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, …

3, 6, 9, 12, 15, 18, 21, 24, 27, …

All of the circled numbers are common multiples, the light blue (first circle) is around the **Lowest** (least) **Common Multiple**. So the LCM(2, 3) = 6. When we working with fractions we use the **LCM** to find common denominators; however, we use the term **Lowest Common Denominator (LCD)** when speaking about denominators.

If we look at all three values, find the LCM(2, 3, 5):

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32. …
3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, …
5, 10, 15, 20, 25, 30, 35, 40, 45, …

Since 30 is the first common factor of the first three prime number the LCM(2, 3, 5) = 30. So $2 \times 3 \times 5 = 30$ which is a fast way to get a common multiple. However, working LCM by this method can be a **trap!** Many have been taught this as a quick way to find a common multiple! Yet, if the numbers are not prime (i.e., *relatively prime*) this will **not** get you the lowest (least) common multiple and you will need to do more reducing of fractions.

Example: LCM(15, 35) The **Trap** some will use: 15 x 35 = 525
15, 30, 45, 60, 75, 90, 105, 120, 135, 150, …

35, 70, 105, 140, 175, …

15 and 35 are not *relatively prime* as they share the factor 5. 525 ÷ 5 = 105

LCM(15, 35) = 105, while this was a simple example due to one common factor, it often gets worse (while $15 \times 35 = 525$ is a common multiple, it is **not** the lowest common factor.)

Using this method, try this one: LCM(14, 15, 18)

14, 28, 42, …, 630, … , 1260, …
15, 30, 45, …, 630, … , 1260, …
18, 36, 54, …, 630, … , 1260, …

For 14, you would need to find the 45^{th} factor of 14 for 630.
For 15, you would need to find the 42^{nd} factor of 15 for 630.
For 18, you would need to find the 35^{th} factor of 18 for 630.

There must be an easier way to find the answer than writing all of the factors of each number. Well there is, it is called **factoring** and finding the **prime factors**. Doing this, we find the factors and prime factors of each number.

	Factors	Prime Factors
14:	{1, 2, 7, 14}	2×7
15:	{1, 3, 5, 15}	3×5
18:	{1, 2, 3, 6, 9, 18}	$2 \times 3 \times 3$

The factor list gave us the GCF(14, 15, 18) = 1. But to find the LCM(14, 15, 18), some would multiply the three original numbers together getting: 3,780 (not the LCM). But should notice that there are two 2s in the three prime factor sets, so using only one 2 is necessary. The value is now 1,890. Which is still too large. As there are two 3s, so use only one 3. We only need to use one of them, so dividing by 3 we get 630. This is the simplest answer. There is an efficient way to do this:

14: $1 \times 2 \qquad\qquad \times 7$
15: $1 \times \qquad 3 \quad \times 5$
18: $1 \times 2 \times 3 \times 3$

LCM: $1 \times 2 \times 3 \times 3 \times 5 \times 7 = 630$

*One is not a **PRIME,** but it is a factor of every value.*
The one, 1, is the only shared value of each of the numbers 15, 15, 18.
Hence, 1 is the only common factor in the GCF(14,15,18).

This method uses one times the **prime factorization** of each of the numbers. Any duplicate numbers are used only once in finding the LCM. A cool side effect of this method, it allows us to find the **Greatest Common Factor (GCF)**. The GCF is the number that is common to all values. In this case, the GCF(14, 15, 18) is 1, since no other number is a factor of all of these three numbers.

The GCF(14, 15) = 1, as both numbers have 1 as a common factor.
The GCF(14, 18) = 2, as both numbers have 2 as a common factor.
The GCF(15, 18) = 3, as both numbers have 3 as a common factor.

Examples using GCF:

Jaime needs to ship 18 comedy DVDs, 30 animae DVDs, and 42 musical DVDs. He can pack only one type of DVD in each box and he must pack the same number of DVDs in each box. What is the greatest number of DVDs Jaime can pack in each box?

Jaime decide to find all of the factors for 18, 30, and 42.

18: {1, 2, 3, 6, 9, 18}

30: {1, 2, 3, 5, 6, 10, 15, 30}

42: {1, 2, 3, 6, 7, 14, 21, 42}

Of the two common factors, the greatest common factor of these is 6, so each box can contain only 6 identical DVDs.

Joey told Jaime, "I have another way to find the GCF using prime numbers."

18: 2 x 3 x 3

30: 2 x 3 x 5

42: 2 x 3 x 7

Jaime said, "Joey, do you notice that all three numbers share 2 x 3 = 6."

The **greatest common factor** of the numerator and denominator is used to reduce fractions to lowest after you have found their sum, difference, product, or quotient.

Finding Prime Numbers

Definition: A **Prime Number** has exactly two *unique* factors.

- The number 1 has only one factor, 1; therefore, it is not a prime number.
- The number 2 has factors of 1 and 2; therefore, it is a prime number.
- The number 3 has factors of 1 and 3; therefore, it is a prime number.
- The number 4 has factors of 1, 2, and 4; therefore, it is not a prime number; it is a composite number.
- The number 5 has factors of 1 and 5; therefore, it is a prime number.
- The number 6 has factors of 1, 2, and 3; therefore, it is not a prime number; it is a composite number.
- The number 7 has factors of 1 and 7; therefore, it is a prime number.

… (See assigned chart in file 01 Translating English to Algebra.)

Continue this process through 100, crossing out the values which are not prime numbers.
https://mathsbot.com/activities/sieveOfEratosthenes

List the Prime Numbers you found:

____

____

Numbers which have multiple factors (3 or more) are call **Composite/Compound Numbers**.

List the Composite/Compound Numbers 1 through 20:

____

____

Find the **Prime Numbers** using this chart to sift them out:
https://mathsbot.com/activities/sieveOfEratosthenes

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Follow these instructions for the chart above to find the list of Primes (multiple-colored highlighters work great here).

1. Place an 'X' over the number 1. (This is because 1 is a factor of every number.)
2. Circle the number 2. (This is the first prime number.)
3. Place an 'X' over every multiple of 2 (or highlight with a highlighter).
4. Circle the next number not crossed out, the number 3.
5. Place an 'X' over every multiple of 3 (or highlight with a highlighter).
6. Circle the next number not crossed out, and
7. Place an 'X' over every multiple of this number (or highlight with a highlighter).
8. Repeat 6 & 7 until you have either a circle or a 'X' on all numbers (or highlight with a highlighter).

You now have a list of the most common Prime Numbers used through most of First Year Algebra (covered by GED tests.)

The following table depicts an interesting theorem about primes 5 or greater. The theorem states that **all primes after the first two are either one more or one less than a factor of 6**. The table below shows the primes highlighted in yellow. A careful examination of the pattern demonstrates all know prime numbers are either before or after a multiple of 6. The chart has six numbers on each row to help demonstrate a proof of the theorem. Notice that not all values on either side of a multiple of 6 is a prime (25, 35, 50, 65, 77, 91, …).

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100	101	102
103	104	105	106	107	108
109	110	111	112	113	114
115	116	117	118	119	120
121	122	123	124	125	126
127	128	129	130	131	132
133	134	135	136	137	138
139	140	141	142	143	144

Interesting just for fun factoids on **prime numbers**:
(These factoids are not on any HSE test.)

1) Twin, cousin, and sexy primes are of the forms (p, p + 2), (p, p + 4), (p, p + 6) respectively, for p a prime. (3 & 5 are twins, 3 & 7 are cousins, 7 & 13 are sexy primes)
2) A prime triplet is a set of three prime numbers of the form (p, p + 2, p + 6) or (p, p + 4, p + 6) (5, 7, & 11 are triplets)
3) So obsessed are some with the magic of prime numbers, sub-categories have been spawned recently—all of which fascinate many on their own.

(p, p + 2)	Twin primes (having a difference of 2)
(p, p + 4)	Cousin primes (have a difference of 4)
(P, p + 6)	Sexy primes (have a difference of 6)
(p, p + 2, p + 6) or (p, p + 4, p + 6)	Prime triplets
(p, p + 6, p + 12)	Sexy prime triplets

Sieve of Eratosthenes https://mathsbot.com/activities/sieveOfEratosthenes
Special Sieve 6 x 14 rows https://www.geogebra.org/m/j4UyPdKW#material/kmzmaw76
GED Book 1 of Lessons: https://www.geogebra.org/m/j4UyPdKW#material/uGX53dy7

Sieve means to **sift**.