

$$(2k)^2 = 4k^2 \leq 4k^2 + 4k = 2k(2k + 2) < 4k^2 + 4k + 1 = (2k + 1)^2$$

$$2k \leq \sqrt{2k(2k + 2)} < 2k + 1$$

$$\sum_{k=1}^n 2k \leq \sum_{k=1}^n \sqrt{2k(2k + 2)} < \sum_{k=1}^n (2k + 1)$$

$$2 \frac{n(n + 1)}{2} \leq \sum_{k=1}^n \sqrt{2k(2k + 2)} < \frac{3 + 2n + 1}{2} \cdot n$$

$$n + 1 \leq \frac{1}{n} \sum_{k=1}^n \sqrt{2k(2k + 2)} < n + 2$$

$$\left\lceil \frac{1}{n} \sum_{k=1}^n \sqrt{2k(2k + 2)} \right\rceil = n + 1$$