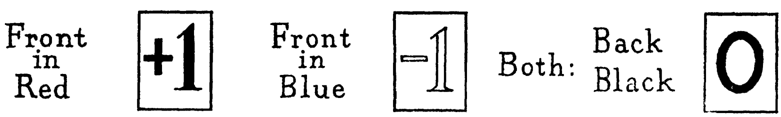
THE ADDITION AND SUBTRACTION OF NEGATIVE NUMBERS

By P. A. Kaner

The usual method of teaching negative numbers is by means of a " number scale " illustrated by thermometers, bank balances and the like—This method suffers from the disadvantage that the properties discussed are those of measurement and not of number. When the important step of abstraction has to be taken and negative numbers handled in their own right, the break is considerable, and many children fail to master the addition and subtraction of negative numbers in (for example) equations and logarithms with negative characteristics.

In order to make convincing the existence of negative numbers in their own right it has been found effective to present positive and negative numbers as " conjugates with respect to zero " starting only from the basis that the sum of two conjugates is zero. The simple apparatus described below has proved most successful in the lower forms of secondary schools and has produced, in a very short time, accuracy in the handling of negative numbers which has surprised the writer himself.

The apparatus required by each child is 24 rectangular pieces of plywood $3'' \times 4''$. (A set of 24 can actually serve more than one child.) Twelve are marked $+1$ on their faces and twelve are marked -1; the backs may be either left blank or marked 0 as the teacher prefers.



When a $(+1)$ face is placed on a (-1) face the result is a zero pair. This is (strangely) understood very easily even by very young children (9–10). The idea of conjugacy is extended by representing $(+5)$ as five $(+1)$ faces and (-5) as five (-1) faces and here again the result of addition is seen to be zero.

The next step is general addition. Some examples will illustrate the method.

e.g. 1. $(+5)$ add $(+3) = (+8)$ by simply laying down five $(+1)$'s and then three $(+1)$'s.

e.g. 2. $(+5)$ add $(-3) = (+2)$ by laying down five $(+1)$'s the three (-1)'s will now be added *on top of* three $(+1)$'s making three zeros and two $(+1)$'s.

e.g. 3. (-5) add $(+3) = (-2)$ The same as above reversing + and −.

e.g. 4. (-5) add $(-3) = (-8)$ By laying down five (-1)'s and then 3 more (-1)'s.

Addition on this basis is so easy that after a little practice almost all children get 100% correct results. For subtraction $(+5)-(+3)$ and $(-5)-(-3)$ are equally easy. In the first case five $(+1)$ faces are laid out and three removed; in the second five (-1) faces are laid out and three removed.

At this stage it can be shown that an alternative to removing the (-1) or $(+1)$ faces is to cancel them with their conjugates i.e. forming zero pairs. In this way it can be shown that "to take away (-3) we add $(+3)$ or that to take away $(+4)$ we add (-4)." But this should in no circumstances be taught as a rule of thumb.

The subtractions which give most difficulty to all children are of the type $(+3)-(+5)$ or $(-3)-(-5)$. To teach the first we lay out in addition to three $(+1)$ faces some "zero pairs". For $(+3)-(+5)$ we remove the three $(+1)$ faces *and also* $(+1)$ *faces from two zero pairs*. This exposes two (-1) faces and gives the answer as (-2). Similarly $(-3)-(-5) = +2$. For $(+3)-(-2)$, we remove the (-1) faces from two zero pairs and altogether five $(+1)$ faces are exposed.

After some practise in this stage most children will quickly see that "instead of subtracting (-1) we can add $(+1)$" and all sums of this type will be quickly done by addition of conjugates; for them, the fact that $(-3)-(-5) = (-3)+(+5)$ provides an acceptable short cut, and there is of course no need to use the apparatus once the principle of "subtraction by addition of conjugates" has been established.

Each child should have his own apparatus, possibly having made it himself and manipulate it himself. When a number of children made their own $+1$ and -1 sets some very attractive sets were produced. The nicest pairs were (i) those painted white on both sides and the $+1$ or -1 marked very boldly in red and blue over the white in gloss paint (ii) those painted yellow on the back and red and blue on the front with the $+1$ and -1 marked over in white.

For many children the handling of the wooden symbols $+1$ and -1 gives them a feeling of mastery over these numbers, a sense of being in control. This contrasts strongly with the feelings of many children when they are taught by the rule "change the sign and add", or by using a number scale.

It is helpful to follow up this work with work on equations in which addition and subtraction of negative numbers is necessary.

Conclusion

This approach has been tried on groups of children varying in ability from top Grammar School to C stream Secondary Modern and has been met with interest and success. It has the great advantage that, as well as providing an easy technique for the addition and subtraction of negatives, it paves the way for discussion of conjugacy in many other forms. For example, the similar type of conjugacy of reciprocals with respect to the operations multiplication and division gives an insight of that internal structure and organization that links mathematics with art and music.

P. A. Kaner

Piedmont,
Scrubs Bottom,
Bisley, Glos.

ADDITION AND SUBTRACTION

By J. S. Higgins

The following is an account of a method of addition and some methods of subtraction which may appeal to teachers who like to lace their teaching with enrichment material, but who sometimes refrain from doing so because of a fear that an " off beat " treatment of a topic may confuse rather than clarify.

Most pupils have so firm a grasp of addition and subtraction that there seems little danger in leading pupils off the highway and down one or two by-ways and advantages may accrue in the way of heightened class interest in Mathematics.

Addition

" Front end " Addition

This method is attributed to the Hindus. It consists of adding the " thousands ", " hundreds ", " tens ", and " units " columns in that order, that is, the reversal of the usual order. The examples given below display the mechanism of the process.

The advantages of the method are that it requires less mental arithmetic and is more accurate in the sense that, since mistakes are more likely to occur in the later stages of a calculation, mistakes, if they occur, will affect the answer to a lesser degree than in the usual " back end " addition.

The chief disadvantage is that the process requires more space than the usual method.

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Author(s): P. A. Kaner

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