This page is **linked** on your computer screen on the day of your Exam. The pages below help explain many of these formulae. Notes in red are designed to assist the learner. Ask Instructor whenever you need help.

2014 GED® Test Resources

Mathematics Formula Sheet & Explanation

The 2014 GED® Mathematical Reasoning test contains a formula sheet, which displays formulas relating to geometric measurement and certain algebra concepts. Formulas are provided to test-takers so that they may focus on *application*, rather than the *memorization*, of formulas.

Area of a:

square	$A = s^2$
rectangle	$A = lw$
parallelogram	$A = bh$
triangle	$A = \frac{1}{2}bh$
trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$
circle	$A = \pi r^2$

Perimeter of a:

square	$P = 4s$
rectangle	$P = 2l + 2w$
triangle	$P = s_1 + s_2 + s_3$
Circumference of a circle	$C = 2\pi r$ OR $C = \pi d$; $\pi \approx 3.14$

Surface area and volume of a:

rectangular prism	$SA = 2lw + 2lh + 2wh$	$V = lwh$
right prism	$SA = ph + 2B$	$V = Bh$
cylinder	$SA = 2\pi rh + 2\pi r^2$	$V = \pi r^2 h$
pyramid	$SA = \frac{1}{2}ps + B$	$V = \frac{1}{3}Bh$
cone	$SA = \pi rs + \pi r^2$	$V = \frac{1}{3}\pi r^2 h$
sphere	$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

(p = perimeter of base with area B; $\pi \approx 3.14$)

Data

mean	mean is equal to the total of the values of a data set, divided by the number of elements in the data set
median	median is the middle value in an odd number of ordered values of a data set, or the mean of the two middle values in an even number of ordered values in a data set

Algebra

slope of a line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
slope-intercept form of the equation of a line	$y = mx + b$
point-slope form of the equation of a line	$y - y_1 = m(x - x_1)$
standard form of a quadratic equation	$y = ax^2 + bx + c$
quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Pythagorean theorem	$a^2 + b^2 = c^2$
simple interest	$I = Prt$ (I = interest, P = principal, r = rate, t = time)
distance formula	$d = rt$
total cost	total cost = (number of units) × (price per unit)

GEDtestingservice.com | GED.com

Contents

The above chart is available at https://ged.com/wp-content/uploads/math_formula_sheet.pdf. Its contents are available when you take your exam. The remainder of this document is an extensive explanation of the formulae on the GED® Exam above including **definitions** and **explanations of the vocabulary and abbreviations** used. Additionally, included with the examples from the formula sheet, there are extensive examples of other formulae seen on the GED Mathematics exam which were studied in earlier grades. There are many formulae which were not included on the GED formula sheet, you are expected to know and be able to use elementary and middle school formulae on the exam.

GED© Formula Chart (official) https://ged.com/wp-content/uploads/math_formula_sheet.pdf **1**
Contents 2
Large letters copy of Formula Chart Official Version 3-4
Formula Chart annotated copy **Enhance Versions** 5-22
Many unspecified formulae are included in this document.
GED© Calculator Guide https://ged.com/wp-content/uploads/calculator_reference_guide.pdf **23**
Geometric Figures
Geometry Basic Definitions 24
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Geometric Solids 27
Properties of Mathematical Sentences 28
Order of Operations { **PE(MD)(AS) or G(ER)(MD)(AS)** } 29-30
Links to Geometry Videos 31

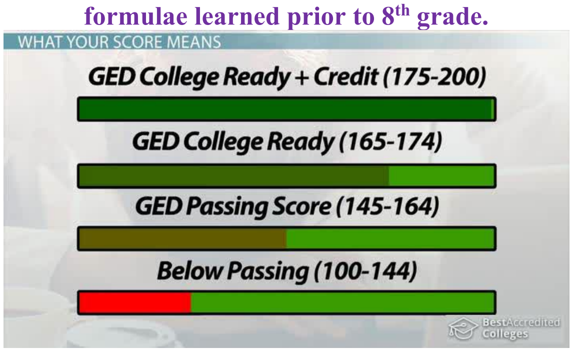
GeoGebra interactive links for GED/HSE mathematics:

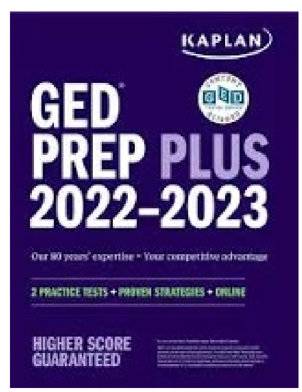
GED Mathematics Book1 https://www.geogebra.org/m/j4UyPdKW
GED Mathematics Book 2 https://www.geogebra.org/m/mEs37yMj

Formula is the singular spelling.
Formulae is the plural spelling.
Adding an –s to formula is a common error made by many English speakers.

Recalling when students are normally introduced to:
Geometry and Number Lines: Kindergarten to present
Addition and Subtraction: K – 2nd grades
1st Informal Intro to Algebra 3 + 🍓 = 8, what is 🍓 ?
Parentheses: 3rd grade to change order of addition/subtraction
Multiplication and Division: 3rd and 4th grades
2nd Informal Intro to Algebra 3 × 🍓 = 18, what is 🍓 ?
Exponents and Roots: 5th and 6th grades
Fractions are ongoing from K, operations within 3rd and 4th
Percentages in 3rd. Decimals: 4th or 5th grade
Signed numbers: 5th or 6th grade; $a - b = a + (-b)$
Variables: 5th grade
Algebra: 6th grade, more on letters as variables; fraction division using reciprocals $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$.

The GED© expects students to recall some of the mathematics formulae learned prior to 8th grade.





References to Kaplan in this document refer to versions of this text starting in 2019, however, www.ged.com is final authority.

College Ready

If you see this, learning above the basics of the topic can lead to higher scores. Basic skills are still needed on the topic.

Mathematics Formula Sheet & Explanations

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Square	$P = 4s$
Rectangle	$P = 2l + 2w$
Triangle	$P = s_1 + s_2 + s_3$

Circumference of a circle

$C = 2\pi r$ OR $C = \pi d$; $\pi \approx 3.14$

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Pythagorean theorem $a^2 + b^2 = c^2$

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(I = interest, P = principal, r = rate, t = time)

distance formula $d = rt$

(d = distance, r = rate of speed, t = time of travel)

total cost total

cost = (number of units) × (price per unit)

Mathematics Formula Sheet & Explanations

All text in **RED** are notes, most text in **BLACK** are from GED to page **18**.

The 2014 GED® Mathematical Reasoning test contains a formula sheet, which displays formulas relating to geometric measurement and certain algebra concepts. Formulas are provided to test-takers so that they may focus on *application*, rather than the *memorization*, of formulas.

Area of a: A surface covered by squares of some size.

Square	$A = s^2$
Rectangle	$A = lw$
Parallelogram	$A = bh$
Triangle	$A = \frac{1}{2}bh$
Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$
Circle	$A = \pi r^2$

Perimeter of a: Perimeter means to find the sum of all sides.

Square	$P = 4s$
Rectangle	$P = 2l + 2w$
Triangle	$P = s_1 + s_2 + s_3$

Circumference of a circle

$$C = 2\pi r \text{ OR } C = \pi d;\ \pi \approx 3.14$$

A Simple Explanation of Area and Perimeter https://schoolyourself.org/learn/geometry/area 12
PHET: Area Builder: https://phet.colorado.edu/en/simulations/area-builder

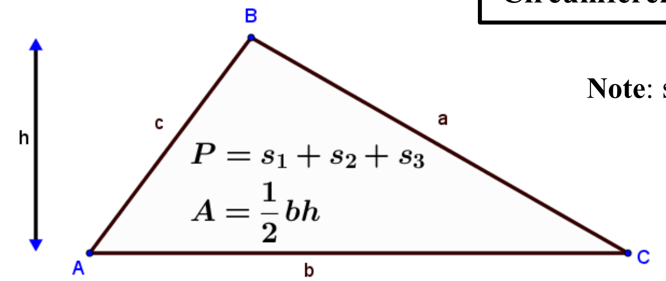
The perimeter of plane figures is found by adding the lengths of all sides to find the total path around a figure. Circumference is how the perimeter of a circle is computed; the circumference of other curved surfaces is not assessed on HSE exams.

Students are expected to be able to use all the formulae on the formula sheet where any single value can be the unknown value within the problem.

Perimeter and Area (Plane Figures) ★★★★★

Triangle

Perimeter means to add the lengths of all sides together.
Circumference is the term for the linear distance of a circle's edge.

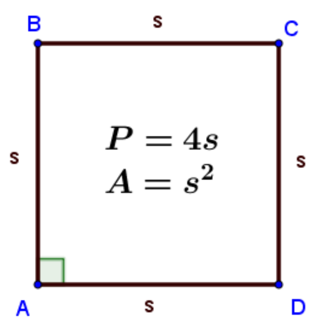


https://www.geogebra.org/m/mEs37yMj#material/erssygnk

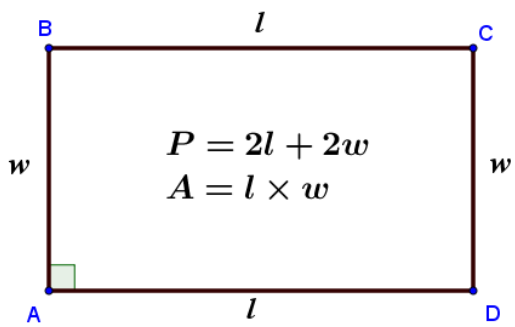
Note: $s_1 + s_2 + s_3 \equiv a + b + c$

The **Height** is always perpendicular (⊥) to the **Base**. Rectangles and squares are special parallelograms whose adjacent sides are perpendicular. The height is the distance between pairs of parallel sides of these quadrilaterals.

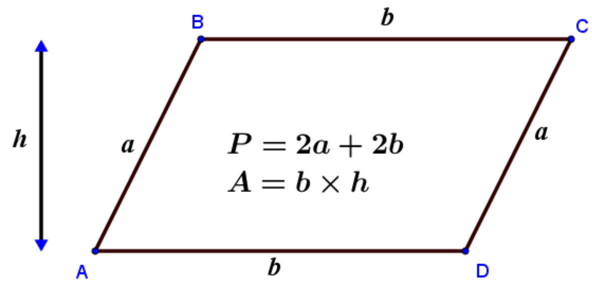
Square



Rectangle



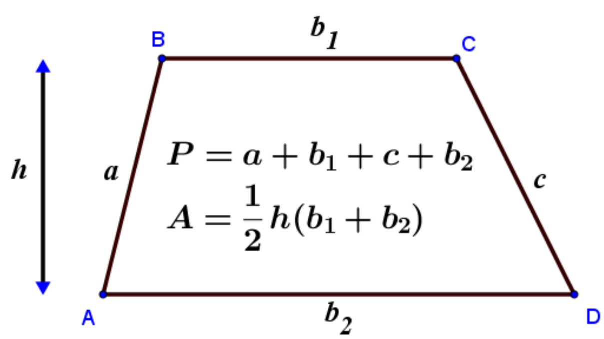
Parallelogram



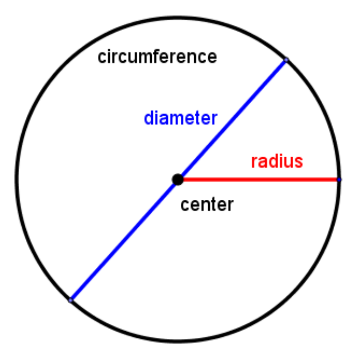
https://www.geogebra.org/m/mEs37yMj#material/eqktdbvc
https://www.geogebra.org/m/mEs37yMj#material/cupdv7xt

A rhombus (rhombi) is a parallelogram with all side congruent, no right angles.

Trapezoid



Circle



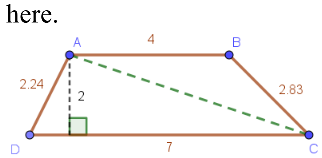
$C = \pi d$

$C = 2\pi r$

$A = \pi r^2$

https://www.geogebra.org/m/ybwtx8jb

In general, the area of all quadrilaterals can be separated by a diagonal from non-adjacent vertices of the quadrilateral forming two triangles. If one can find the height and a base of each triangle, the area of the quadrilateral is the sum of these two triangles. An example is the trapezoid shown here.



$\triangle ACD = \frac{1}{2}b_1 h = \frac{1}{2} \bullet 7 \bullet 2 = 7$

$\triangle ABC = \frac{1}{2}b_2 h = \frac{1}{2} \bullet 4 \bullet 2 = 4$

$A = \frac{1}{2}(b_1 + b_2)h = 7 + 4 = 11$

Note: Many times it is recommend setting $\pi = 3.14$, this will give the exact value shown as the answer choice. However, the π key of the calculator will give a more accurate answer slightly higher decimal result usually with the first 3 digits being the same. Yet, this should not be a problem for a student selecting the correct solution, as the beginning 3 digits will be identical and with rounding nearly the same value.
If solution is to be typed, one uses the 3.14 as advised.

Geometric Vocabulary List

Angle: the corner formed by two sides of a plane figure
Area: the space enclosed by a plane figure; a count of the number squares on the surface
Circle: any plane closed shape/figure the same distance from the center
  Semicircle: half of a circle
Congruent: the parts of a geometric figure which are exactly equal; ≅.
Concave polygon: at least one interior angle is more than 180 degrees
Convex polygon: all interior angles are less than 180 degrees
Diagonal: a line connecting two non-consecutive vertices in a plane or solid figure.
Edge: the line connecting adjacent points in a solid figure
Face: the surface of a solid figure.
Line segment: a segment between any 2 points
Parallel lines: lines in a plane that never intersect (cross)
Perimeter: the distance around a plane figure; add all sides
  Circumference: this distance around a circle (curved figure)
Plane figure: a geometric figure in a plane (flat) surface
Quadrilateral: any plane closed figure/shape with four straight sides
Parallelogram: a quadrilateral with opposite sides parallel and congruent
  Rectangle: a parallelogram with right angles
  Rhombus: a parallelogram with all congruent sides   **Square**: a rectangle with congruent angles and sides
Trapezoid: a quadrilateral with only one pair of parallel sides
  Isosceles Trapezoid: a trapezoid whose non-parallel sides and base angles are congruent
Polygon: a plane figure with three or more sides and angles
  Regular Polygons: convex polygons with congruent sides and angles
Polyhedron: A 3-dimensional solid made by joining polygons
Similar: The shapes with same angles, numbers of sides, different lengths
Solid: a 3-dimensional (3D) figure, such as sphere, cone, prism (box), pyramid
Sphere: a circle rotated about its diameter
Triangle: Any plane closed figure/shape with three sides
  Equilateral Triangle: a triangle with all sides and angles equal
  Isosceles Triangle: a triangle with two congruent sides (≅) with the angles opposite the ≅ sides are congruent (≅).
Vertex: the point where 2 or more lines intersect
Volume: the amount of space inside of a solid; count the cubes that fill the solid

Formula Abbreviations:

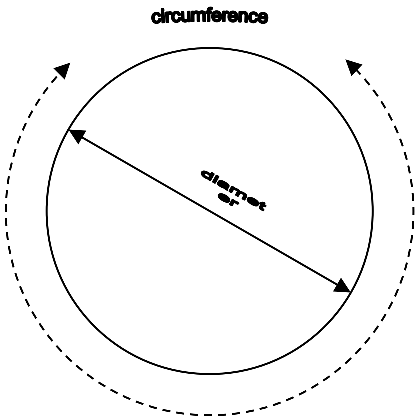
- **A**: <u>Area</u> of a plane figure; **B**: Area of the **B**ase face of a solid figure
- **C**: <u>Circumference</u> of circle;   **r**: <u>**r**adius</u> of circle or sphere
- **d**: <u>**d**iameter</u> of circle or sphere; **d = 2r**
- **P**: <u>perimeter</u> of a plane figure; or **p**: perimeter of base of are prism
- **h**: <u>**h**eight</u>; **l**: <u>**l**ength</u>; **w**: <u>**w**idth</u>; **d**: <u>**d**epth</u>
- **s**: <u>side</u> on a square; or ***s***: <u>*slant height*</u> on the polygons or cone's side (you may need to use the Pythagorean to computer parts of the triangle.)
- **SA**: <u>Surface Area</u> of a solid figure
- **V**: <u>Volume</u>

<u>The Number Sets</u>: These are the common names of sets which can be used in instructions and for solutions.

- **counting—** the numbers we count or enumerate by: {1, 2, 3, 4, … , 1001, 1002, …}
- **whole numbers—** 0 and counting numbers: {0, 1, 2, 3, 4, … , 1001, 1002, …}:
- **integers—** the negatives of the counting and the whole numbers: {…, –3, –2, –1, 0, 1, 2, 3, …}
- **rational numbers—** any number that can be written as a fraction: $\frac{any\ integer}{any\ non-zero\ integer}$. All terminating and repeating decimals can be represented by a ratio (a fraction).
- **irrational numbers—** $\{\sqrt{2}, \sqrt{3}, \sqrt{5}, \dots, \sqrt{8}, \pi, \sqrt{10}, \dots\}$ numbers whose decimal has no patterns, not be rational.
- **real numbers—** all of the above sets combined https://www.geogebra.org/m/j4UyPdKW#material/Vv5cQRBB
- **non-Zero numbers—**all numbers whose value is not ZERO, 0. Think also of 'non-negative' and 'non-positive'

CIRCLES

Circumference and Area



$$\pi = \frac{\text{CIRCUMFERENCE}}{\text{DIAMETER}}$$

π = 3.14159265…

π (pi) is an irrational number.

Irrational numbers continue forever with no fixed patterns.

Rational approximations in use:

$\frac{22}{7}$ = **3.142857** is used in middle schools.

$\frac{355}{113}$ = **3.14159292** ... was used by the **Greeks.**

Using the π key on the calculator is more accurate, so answers will be more accurate. You can use the π key answer will not match using 3.14 exactly, but the answers will be indistinguishable in multiple choice solutions only. If fill-in-blank, use 3.14 as stated in the problem.

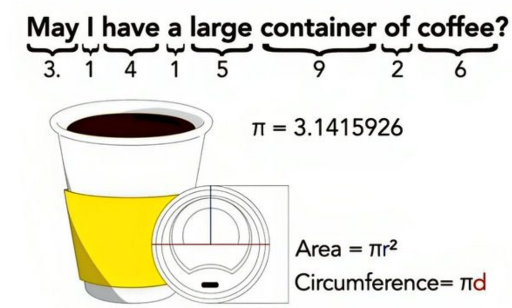
C = 2πr OR C = πd; π ≈ 3.14

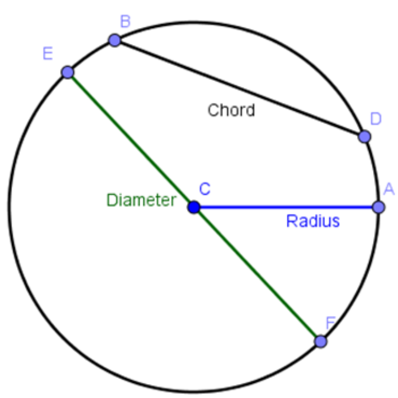
circumference = π × diameter

or

circumference =2 × π × radius

What is Pi? https://www.youtube.com/watch?v=cC0fZ_lkFpQ



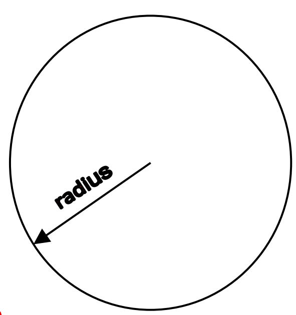


← Parts of a circle students should know…

area = π r^2

or

area = π × radius × radius



Radius is singular, **radii** is the plural.

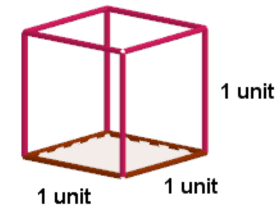
Cherry Pie is Delicious

Apple Pies R 2

Interactive calculations: https://www.geogebra.org/m/ybwtx8jb

Surface Area is the count of squares covering a surface, the cube on the right has a SA = 4 square units. Surface area is called the **Net** the solid figure.

Volume is a count of the cubes within a 3D solid, the cube on the right is one cubic unit.



The volume of a cube 1 × 1 × 1 is 1 cubic unit.

Surface area and volume of a:

https://www.geogebra.org/m/mEs37yMj#chapter/1067827 **3D Solids Interactive Lessons**

| Lateral Area of Sides |

Rectangular prism $SA = 2lw + 2lh + 2wh$ $V = lwh$

bottom/top + front/back + left/right side

right prism $SA = ph + 2B$ $V = Bh$

lateral + top/bottom

Lateral area is the area of all the sides without top and bottom area; '***ph***' is the lateral area.

https://www.geogebra.org/m/gU22RUUA

cylinder $SA = 2\pi rh + 2\pi r^2$ $V = \pi r^2 h$

label area + top/bottom

https://www.geogebra.org/m/mEs37yMj#material/nzbdbykm

pyramid $SA = \frac{1}{2}ps + B$ $V = \frac{1}{3}Bh$

sides + base

The base, **B**, of a pyramid can be any (regular) polygon, slant height, **s**.

https://www.geogebra.org/m/mEs37yMj#material/qdrdn5rn

cone $SA = \pi rs + \pi r^2$ $V = \frac{1}{3}\pi r^2 h$

cone + cap

https://www.geogebra.org/m/mEs37yMj#material/tzwn2k6e

sphere $SA = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

https://www.geogebra.org/m/mEs37yMj#material/nsgex9aj

(p = perimeter of the base with an area of B; $\pi \approx 3.14$)

Recall: Area of a circle is πr^2.

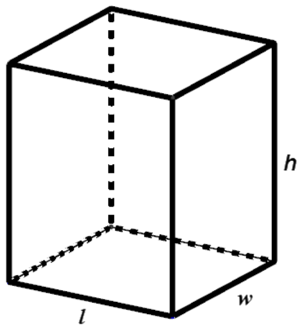
The circumference is $2\pi r$.

The perimeter of a rectangle is $2l + 2w$.

Also, when doing multiple choice question, π on the calculator keyboard is faster than 3.14. If you must type the answer, then use 3.14 and check where to round off decimal values.

A regular polygon has all sides equal in length. Irregular polygons are not tested on GED/HSE exams.

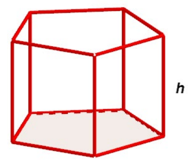
Rectangular Prism

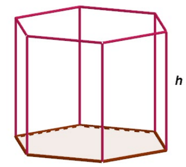


$SA = 2\,lw + 2\,lh + 2\,wh$

$V = lwh$

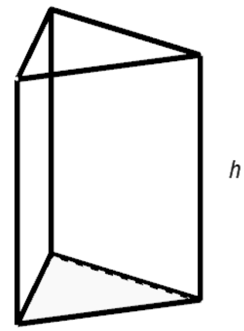
Right Prisms





Other **Right Prisms** above

Right Prisms have rectangular sides. The **Base** of a right prism can be any (**regular**) polygon. The perimeter, ***p***, of a regular polygon is (length of the side) × (number of sides). HSE tests will give Base for any shape other than a square or a triangle.

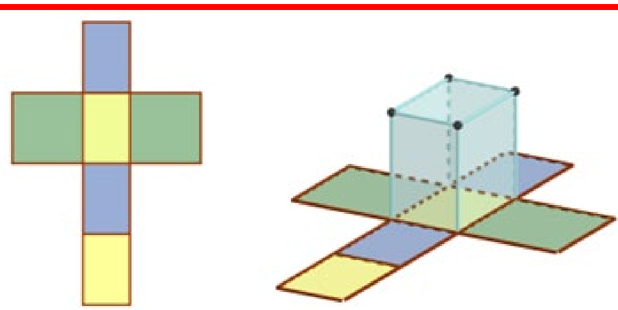


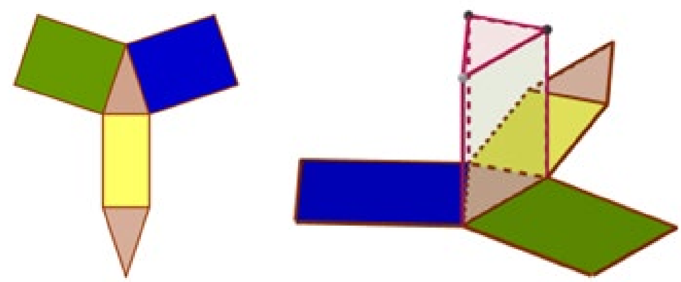
$SA = ph + 2B$

$V = Bh$

https://www.geogebra.org/m/mEs37yMj#material/dd8bdhsg

Net of prism





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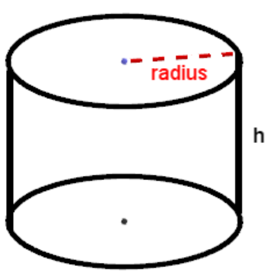
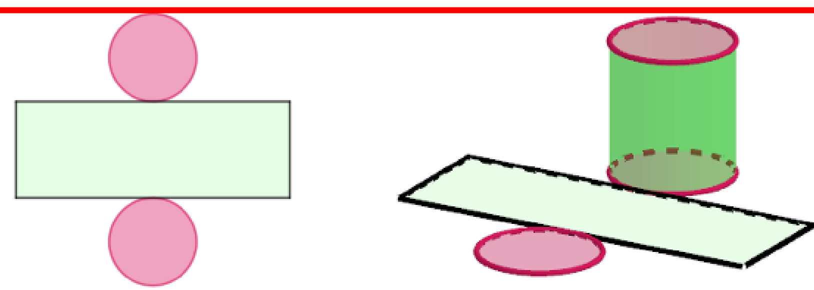
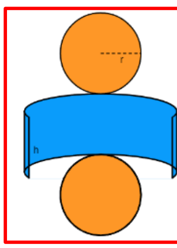


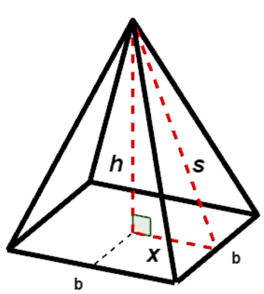
$SA = 2\pi rh + 2\pi r^2$

$V = \pi r^2 h$



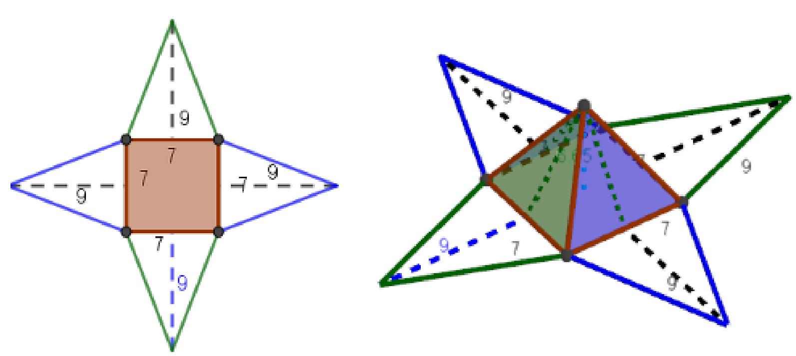


https://www.geogebra.org/m/mEs37yMj#material/nzbdbykm



$SA = \frac{1}{2}ps + B$

$V = \frac{1}{3}Bh$



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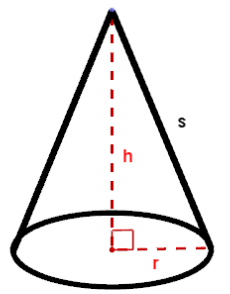
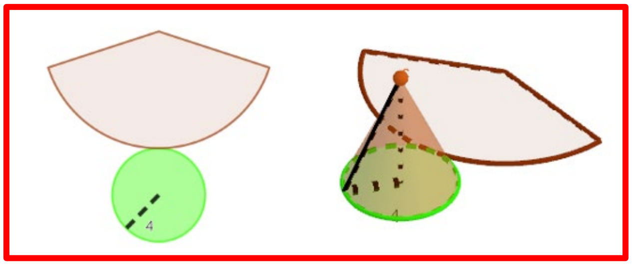


$SA = \pi r s + \pi r^2$

$V = \frac{1}{3}\pi r^2 h$



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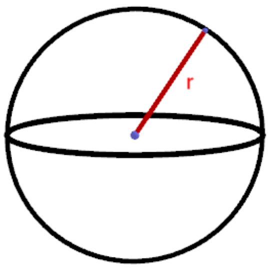
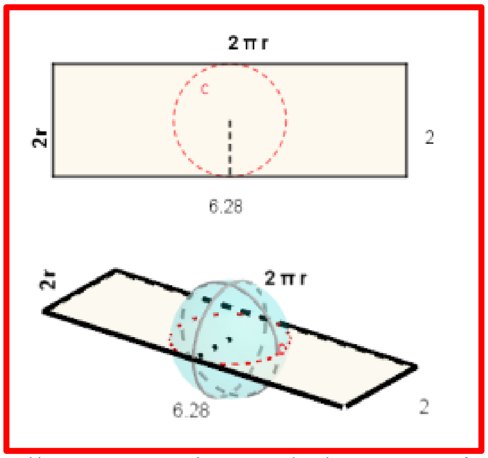


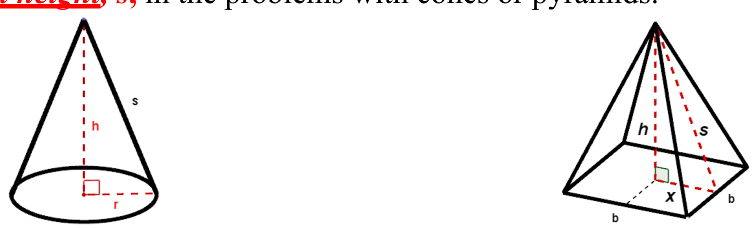
$SA = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$



https://www.geogebra.org/m/mEs37yMj#material/nsgex9aj
https://www.geogebra.org/m/mEs37yMj#material/gwwqf47z animation
https://www.geogebra.org/m/mEs37yMj#material/p4jd88uu

For many problems, the Pythagorean Theorem is needed to find the ***height, h***, value of ***r*** or ***x***, or ***slant height, s,*** in the problems with cones or pyramids.



Pythagorean Theorem: $a^2 + b^2 = c^2$

height: h slant height: s radius: r — x: the distance from center to side

$x = \frac{1}{2}b$

$r^2 + h^2 = s^2$ — $x^2 + h^2 = s^2$ or $(\frac{1}{2}b)^2 + h^2 = s^2$

Solve for the missing value.

College Ready

https://www.geogebra.org/m/mEs37yMj#material/uTvy5sKR

Data (Analysis) Mean, median, mode, and range are on the Social Studies and Science Exams, also.

- **mean** is equal to the total of the values of a data set, divided by the number of elements in the data set.
  Missing value using average: $Desired\ average = \frac{Sum\ of\ known\ values + the\ unknow}{number\ of\ value}$; 90 = (450 + x)/6, solve for x = 90.

  Sorting all values from low to high or high to low will assist for the following three values in a list.
- **median** is the middle value in an odd number of ordered values of a data set, or the mean of the two middle values in an even number of ordered values in a data set. {Remember: 2 possible methods.}
- **mode** is the number which occurs the most frequently in a list of numbers. There can be several answers to this situation: no mode (none), a single mode, or multiple modes. {Never answer with a '0', unless '0 is' a set member and the mode of the set of values}

Example 1: {1, 2, 4, 6, 7, 10} Mean = 5, Median = $\frac{4+6}{2}$ = 5, Mode: None, Range = 9 Example 2: {1, 3, 5, 7, 9} Mean = 5, Median = 5, Mode = None, Range = 8 Example 3: {}	Unless a 0 is in the set, and 0 cannot be the actual mode. Example 1: {-1, -1, 0, 0, 0, 1, 1} odd list of elements Example 2: {-1, -1, 0, 0, 0, 0, 1, 1} even list of elements Example 3: {-1, -1, 0, 0, 1, 1} an example of a triple mode; median, mean, range All 3 have a: Mean = 0, Mode = 0, Median = 0, Range = 2

- **range** is the difference between the largest and smallest values of a set of numbers.
  Practice applet: https://www.geogebra.org/m/mEs37yMj#material/ZJztkKaz

For **median**, **mode**, and **range** order the values in the list.

The **median** is the middle value. Since there can be an even or an odd number of elements in a list, there two ways to find the median.

1) Locate the center of the list.
2) a) If the list has an odd number of elements, middle value is the median.
   b) For a list of an even number of elements, if the middle two elements are the same that is the median;
   c) otherwise, you must add those two elements and divide by 2 for the median.

The **mode**, **M**ost **O**ften **D**emonstrated **E**lement, can have three possible answers:

1. There are no repeated values--> **NO MODE** or None
2. If there is only one number repeated more than once, this is the mode.
3. If more than one number is repeat the same number of times, there are multiple mode.

The **range** is the difference between the highest and lowest values. So you just need to subtract them. It is a distance, hence there is no sign on the number.

MEAN — AVERAGE
MIDDLE — MEDIAN
MOST OFTEN — MODE
|← RANGE →|

$$Weighted\ Average = Ave1 \times Weight1 + Ave2 \times Weight2 + \cdots + AveN \times WeightN$$

https://www.youtube.com/watch?v=slFqL86q3EA

Find the missing grade needed for **Average** = $\frac{\mathbf{Sum\_Of\_Grades + Missing\_Grade}}{\mathbf{Total\ Number\_Of\_Grades}}$

Mean or average, add the numbers and divide the number of values

$$mean = \frac{Sum\ of\ Values}{Number\ of\ Values}$$

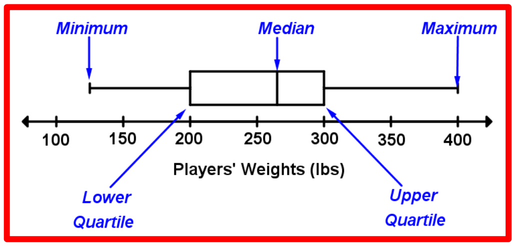
For the next three place all numbers are placed in order from least to highest or highest to lowest.

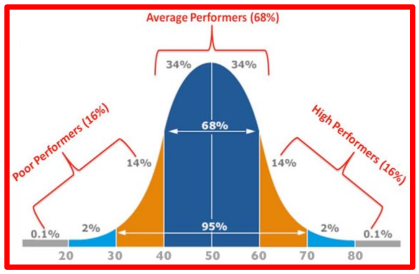
Median--middle value, two ways to get than answers
an odd number values; it is the middle value…
an even number of values; the two middle values are averaged: $\frac{a+b}{2}$

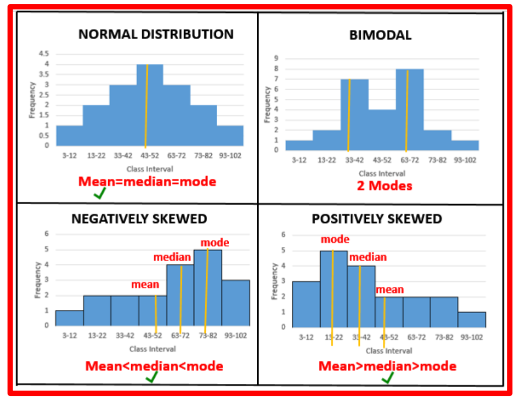
Mode--the number that is repeated the most:
If no number is repeated, the mode is **NO MODE** or **NONE**.
If there is a single mode, it will be value that repeated the most
If several numbers are repeated the same number of times: multiple modes exist…

Range--high value minus the low value

Box and Whiskers Plot | **Bell-Shaped Curve[1]** | **Histogram[2]**







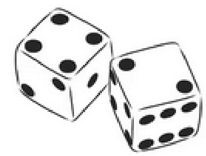
The bell-shaped curve shows the 'mean' at its peak, while the box and whisker plot show the maximum, minimum, range, median, and quartile breaks. A histogram can show the mode(s), median, mean, and range.

[1]https://teacherhead.com/2013/07/17/assessment-standards-and-the-bell-curve/

[2]**StudyWalk** website; **(Kaplan, pp. 280-295)**

https://www.youtube.com/results?search_query=howie+hua+math+permutation+and+combination+formulas

Probability (Kaplan, pp. 296-299)



A pair of Dice

Single events or occurrences {examples using a pair of dice shown}

$$P(A) = \frac{number\ of\ incidences}{number\ possibilities};\ P(7) = \frac{6}{36} = \frac{1}{6};\ \ P(>7) = \frac{15}{36} = \frac{5}{12};\ P(<5) = \frac{6}{36} = \frac{1}{6}$$

Independent Events If two events, *A* and *B* are independent then the joint probability is

P(A and B) = P(A ∩ B) = P(A)P(B) → $P(A) \times P(B) \ldots \times P(C)$

Two events are independent if the occurrence of one event does not affect the chances of the occurrence of the other event, i.e., coin tosses, sex of a child, roll of a die or dice. (See Essential Ed, Probability Lesson, Level D)

A Die

Sum of Dice Roll						
	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Face Values of Dice on a single Roll						
	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Dependent events where the outcome of one event affects the probability of the next event, i.e., the repeated drawing of colored marbles from a bag, repeated drawing from a deck of cards without replacing the item is: **P(ace first, then king) = (4/52) × (4/51) = 16/2652 = 4/663**

Mutually exclusive events

- If two events are mutually exclusive, then the probability of *both* occurring is denoted as

  P(A and B) = P(A ∩ B) = P(A)P(B)

  - For example, if two coins are flipped, then the chance of both being heads is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.
- If two events are mutually exclusive, then the probability of *either* occurring is denoted as

  P(A ∪ B) = P(A or B) = P(A) + P(B) - P(A ∩ B) = P(A) + P(B) – 0 = P(A) + P(B)

  - For example, the chance of rolling a 1 or 2 on a six-sided die is

    P(1 or 2) = P(1) + P(2) = $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$.
- The complement of probabilities is 1 – P(Event)
- If either event *A* or event *B* can occur but never both simultaneously, then they are called mutually exclusive events. Here are several of the videos chained together, best not to binge watch them.
  - https://www.youtube.com/watch?v=X6usGgwXFyU 11 Minutes
  - https://www.youtube.com/watch?v=94AmzeR9n2w 10 minutes
  - https://www.youtube.com/watch?v=EHU6pVSczb4 18 minutes
  - https://www.youtube.com/watch?v=EHU6pVSczb4 7 minutes
  - https://www.youtube.com/watch?v=EHU6pVSczb4 19 minutes
  - https://www.geogebra.org/m/tBBawAte has some examples withing Examples of Lesson Activities or Game.

There are several built-in calculator probability functions using the [prb] key and the [^] key, most should be familiar with basics. The following website discusses these functions, which are included here:

Permutations and Combinations

College Ready

Illustrating Permutation and Combination - What are the key words? What's their difference?
https://www.youtube.com/watch?v=yIEtTkVpRMY

Permutations (**order matters**)

1) Number of Permutations of n objects—**no repetition allowed**; n [prb] [3]
$$n! = n \times (n-1)! \times (n-2)! \ldots \times 2 \times 1$$
2) Number of Permutation of r objects—**repetition allowed and order matters**; n [^] r
$$n^r$$
3) Number of Permutations of r objects from n objects—**no repetition allowed**; n [prb] [1] r
$$nPr = \frac{n!}{(n-r)!}$$

In combinatorics, 0! represents the number of ways to arrange 0 objects. There is exactly 1 way to arrange 0 objects, and it is doing nothing by this logic

0! = 1

Combinations (**order does not matter**)

1) Number of Combinations of r objects chosen from n objects—**repetition not allowed**; n [prb] [2] r
$$nCr = \frac{n!}{r!\,(n-r)!}$$
2) Number of Combinations of r objects chosen from n objects—repetition allowed; not tested.*
$$nCr = \frac{(n+r-1)!}{r!\,(n-1)!}$$
(Kaplan, pp. 300-305)
https://www.youtube.com/watch?v=2FW5EmJcDQk
3) Factorials count *all possible arrangements* **n** items.

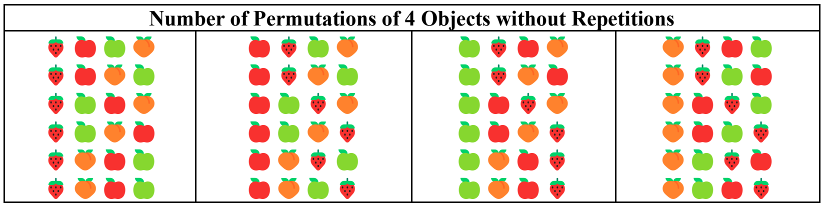
All of the above formulae are available on the TI-30XS calculator. The [prb] keys yield [1] nPr, [2] nCr, [3] !.
n^r is provided by n [^] r, where n the number object one has and r the number objects of interest.

Name		Formula	Example	
Number of Elements		n		
Total Permutations ✎ Without Repetitions		n!	4! = 24 4 [prb] [3] = 24	
Elements to arrange		r	3	2
Permutations ✎ Without repetitions	Order Matters	$\frac{n!}{(n-r)!}$	$\frac{4!}{(4-3)!} = \frac{24}{(1)!} = 24$	$\frac{4!}{(4-2)!} = \frac{24}{(2)!} = 12$
		nPr ✎	4 [prb] [1] 3 = 24	4 [prb] [1] 2 = 12
Permutations ✎ With repetitions		n^r	$4^3 = 64$ 4 [^] 3 = 64	$4^2 = 16$ 4 [^] 2 = 64
Combination ✎ Without repetitions	No order	$\frac{n!}{r!\,(n-r)!}$	$\frac{4!}{3!\,(4-3)!} = \frac{24}{6(1)!} = 4$	$\frac{4!}{2!\,(4-2)!} = \frac{24}{2(2)!} = 6$
		nCr ✎	4 [prb] [2] 3 = 4	4 [prb] [2] 2= 6
Combination With repetitions*		$\frac{(n+r-1)!}{r!\,(n-1)!}$	$\frac{(4+3-1)!}{3!\,(4-1)!} = \frac{6!}{6(3)!} = \frac{720}{36} = 20$	$\frac{(4+2-1)!}{2!\,(4-1)!} = \frac{5!}{2(3)!} = \frac{120}{12} = 10$

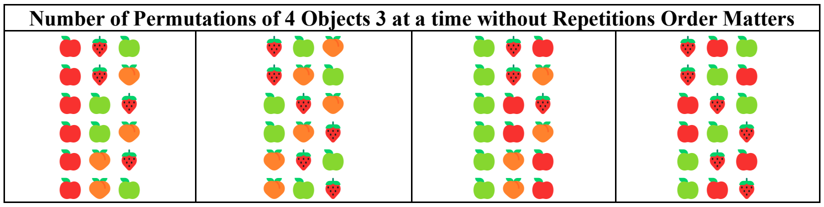
Understanding the permutation and combination formulas with Howie Hua (good videos and explanations)
https://www.youtube.com/results?search_query=howie+hua+math+permutation+and+combination+formulas

*This item is not tested on the HSE examinations. ✎ Formulae list under the [prb] key of your calculator.

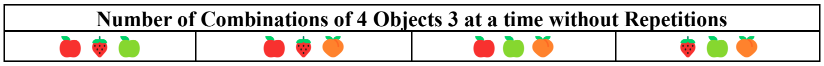
Number of Permutations of 4 Objects without Repetitions



Number of Permutations of 4 Objects 3 at a time without Repetitions Order Matters



Number of Combinations of 4 Objects 3 at a time without Repetitions



Fool-proof method to differentiate between Permutation and Combination

Key takeaways from the article

https://gmatclub.com/forum/learn-structured-approach-to-identify-permutation-combination-questi-263129.html#

- Always keep an eye on the keywords used in the question. The keywords can help you get the answer easily.
- The keywords like- selection, choose, pick, and combination- indicates that it is a combination question.
- The keywords like- arrangement, ordered, unique- indicates that it is a permutation question.
- If keywords are not given, then visualize the scenario presented in the question and then think in terms of combination and arrangement.

Simplifying Expressions (a prerequisite for solving equations and inequalities)

Which of the following operation can be performed: 3🍏 + 4🍓, 3🍏 +4🍏, or 3🍓 + 2🍓 - 5🍏?

Why can you add some, but not all of them? ______________________________

Only items that are alike can be added or subtracted. It does not matter if it is different fruit or variable (letters in the alphabet.) So 3x + 4y cannot be added since x and y are different, but 3x + 4x can be added to be 7x. They are like terms (fruits).
The basic rules for adding and subtracting like terms are the variable parts of a term must be **identical**. If the terms are different, we cannot add or subtract them. The rules for multiplication and division are very different. They will be discussed once you have learned about linear expressions and equations.

The goal for simplifying any expression is to combine all **like terms**, a simplified expression is one where all the terms in the expression have all coefficients combined for each unique variable term.

3x + 5 – 6x – 8 + 12x + 31 since there is only addition and subtraction, we can group like terms keeping signs.
3x – 6x + 12x + 5 – 8 + 31, 3x – 6x + 12x is 9x, 5 – 8 + 31 is 28, resulting in 9x + 28.
3x + 5y + 7 – 6y – 3 + 5x, 3x + 5x + 5y – 6y + 7 – 3, result is 8x – y + 4.

The above are simple examples of linear expressions. On careful examination of the problems, you will find that the reorganization of the terms kept the sign of the original term. At first this may seem like a violation of the Order of Operations, but it is a feature of working with positive and negative numbers: 5 – 4 = 5 + (-4)

While the above were simple examples of **linear expressions**, in essence quadratic, cubic, and other expression follow the exact same rules.

Basic Linear Equations in One Variable★★★★★ https://www.geogebra.org/m/mEs37yMj#material/fjekqnuh & following app.

Whenever an equal sign is placed between two linear expressions, the result is a **Linear Equation**. (Sometimes, a step is not needed.)
Rules for simplifying linear equations in one variable:

1. Simplify each side's linear expression. If you do this with single variable equation (inequalities), set yourself up for 2 to 4 initial addition or subtract choice(s) by doing the opposite of the indicated operation. {a x + b = c x + d, i.e., -a x, -b, -c x, and -d.}
2. If the any the values of a, b, c, or d equals a zero, this reduces the original choices by one. However, if both sides had variables it is works out to be the 2 constant choices.
3. If you have a variable value on both side, either Add or Subtract the variable term so that the resulting variable part on either side of the equation has a positive coefficient.
4. Now, Add or Subtract the constant term with the variable part to both sides of the equation. The constant part is now alone of the other side of the equation.
5. If the variable term has a coefficient different from 1, multiply or divide both sides by the coefficient.
6. The result is the value of the variable..

5x + 15 – 2x = 14 – 8x – 7
3x + 15 = 7 – 8x
{-3x, -15, -7, or +8x}
3x + 15 + 8x = 7 – 8x + 8x
11x + 15 = 7
11x + 15 – 15 = 7 – 15
11x = –8

$$\frac{11}{11}x = \frac{-8}{11}$$

$$x = -\frac{8}{11}$$

Not all lines above need to be written for every problem. 4th and 6th optional

Evaluating Linear Expression and Equation (for formulas and evalating expression)
3x + 5, when x = 7: 3(7) + 5 = 26
3x + 5, when x = 3: 3(3) + 5 = 14

Area of a triangle with base = 6 and height = 8, $A = \frac{1}{2}bh; A = \frac{1}{2} \times 6 \times 8 = 24$

You must be able to find the **base** knowing the A = 24 and the height = 8. $\frac{1}{2} \times b \times 8 = 24$

Basic Linear Equations in Two Variables★★★★★

Forms of Linear Equation

General Form: ax + by + c = 0, where a, b, and c are real constants.

Standard Form: Ax + By = C where A > 0, B, and C are integers {… -2, -1, 0, 1, 2, …}

Slope-Intercept Form: y = mx + b where m is the slope of the line and b is the y-intercept.

https://www.nagwa.com/en/explainers/462136171745/

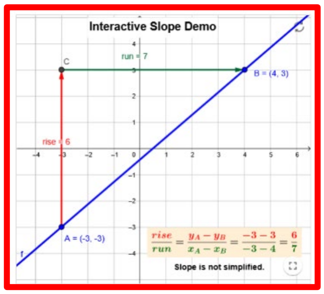
Whenever a linear equation has two variables, the equation can be simplified into several basic forms depending on the use intended for them. The linear equations at the right are the most common forms of linear equations. The slope-intercept form is commonly used on the HSE exams for multiple and various questions. There are two additional formulas, one is the slope, and the other is the Point-Slope form which is designed to assist in getting the value of m when you have two points and b when you only or a slope and single point.

Using the process of solving simple linear equations in one variable, modify it to solve for the one of the two variables in the equation, usually the 'y'.

The x-intercept is where a line crosses the x-axis when y = 0, i.e., (x, 0). {Crucial information for non-linear equations.}
The y-intercept is where a line crosses the y-axis when x = 0, i.e., (0, y). {Crucial information for linear equations.}
The (x, y) coordinates form locations on the coordinate grid. The x-value is for the horizontal axis component, and the y-value is for the vertical axis component.

Finding the **slope** using a graph (1-4) and using a formula (5):

1. Start from the leftmost point https://www.geogebra.org/m/mEs37yMj#material/nqeb3fee
2. Move vertically from that point to the second point (either up or down) **rise**
3. Move horizontally to the right point. **run**
4. The result is the slope of the line. **Rise** : **Run**
5. $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$



Slope-Intercept Methods (Interactive Help) https://www.geogebra.org/m/mEs37yMj#material/hp44vxvk

Solving linear equations:

1. Simplify each side of the equation, i.e., gather the like terms on each side.
2. Isolate the y-term. (Add or Subtract either the x-term and/or the constant, as needed.)
3. Simplify such that the y-term is positive, and so the y-term is by itself on one side of the equation.
4. Isolate the y-variable. (Multiply or Divide each term of the equation by the coefficient of y.)
5. Simplify, such that the equation looks similar to this: y = an expression. (y = m x + b)
6. Use the expression to find the points.

Material from Mr. A. B. Cron's GeoGebra GED Math Book 2 website (interactive apps)

Mean, Median, Mode, and Range https://www.geogebra.org/m/mEs37yMj#material/ZJztkKaz

Basic Terms in Algebra: https://www.geogebra.org/m/mEs37yMj#material/KKBzqa5G

Verbal to Algebraic: Translating: https://www.geogebra.org/m/mEs37yMj#material/g82wNuXT
Worded Expressions: https://mathsbot.com/activities/wordedExpressions
Writing Algebraic Equations: https://www.geogebra.org/m/mEs37yMj#material/zvqq6dek
Visualizing Algebraic Equations Using a Balance Beam: https://www.geogebra.org/m/mEs37yMj#material/fjekqnuh
Writing Word Problems for One-Step Inequalities: https://www.geogebra.org/m/mEs37yMj#material/ehb88cec
Translating and Solving Real World Inequalities: https://www.geogebra.org/m/mEs37yMj#material/wv79hecy
Intro to Linear Equation: https://www.geogebra.org/m/mEs37yMj#chapter/736795
Intro to Quadratic Equations: https://www.geogebra.org/m/mEs37yMj#chapter/737388
Geometry: https://www.geogebra.org/m/mEs37yMj#chapter/87199

Algebra

Points (x_1, y_1) and (x_2, y_2) are interchangeable. **{You have to <u>RISE</u> before you can <u>RUN</u>.}**

slope (m) of a line $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{rise}{run}$ **(Kaplan, pp. 366-375)**

slope-intercept form of the equation of a line: $y = mx + b$

x and **y** are variables; **m** is slope and **b** is the y-intercept value, the coordinate is (0, b).

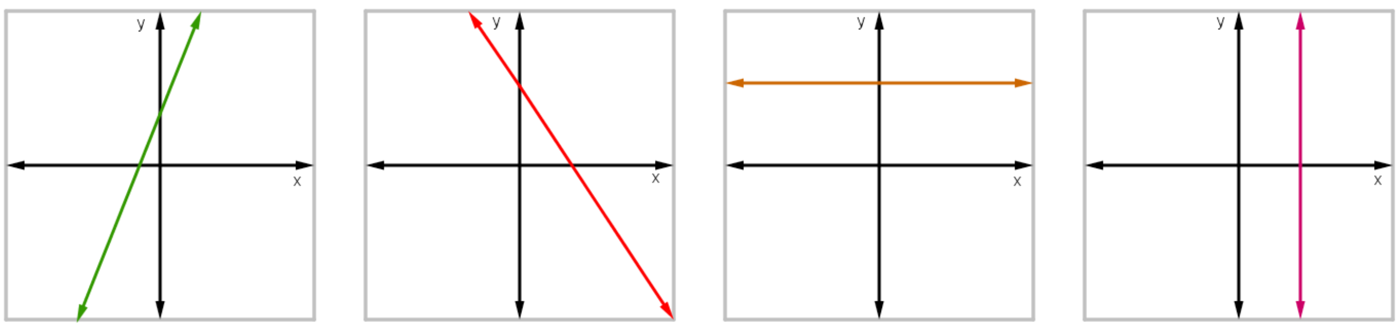
point-slope form of the equation of a line: $y - y_1 = m(x - x_1)$

(x_1, y_1) is any point on the line; **m** is the slope of the line.

The x is any value on the x-axis. The 'b' is the y-intercept's value of the (0, b).

This chart illustrates the characteristics of the four different linear equations possible. **Know it well.**

Postive Slope, m > 0 x- & y-intercepts Rises from left-right	Negative Slope, m < 0 x- & y-intercepts Falls from left-right.	Zero Slope, m = 0 Only y-intercept Line is horizontal.	Undefined Slope (m is undefined) Only x-intercept Vertical Line



Standard Equation Form: $Ax + By = C$ General Equation Form: $Ax + By + C = 0$

Slope-intercept Form: $y = mx + b$, $m = \frac{y_2 - y_1}{x_2 - x_1}$, b is y-intercept with coordinate (0,b)

Point-Slope form: $y - y_1 = m(x - x_1)$

Slope-Intercept Methods (Interactive Help) https://www.geogebra.org/m/mEs37yMj#material/hp44vxvk

Linear **Function Tables** are used to create points on a linear graph; a minimum of 3 points are required (insurance > 3).

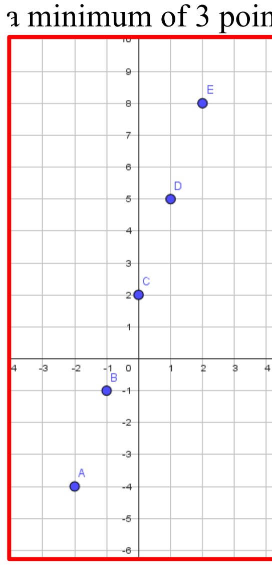
Example: $3x - y = -2$

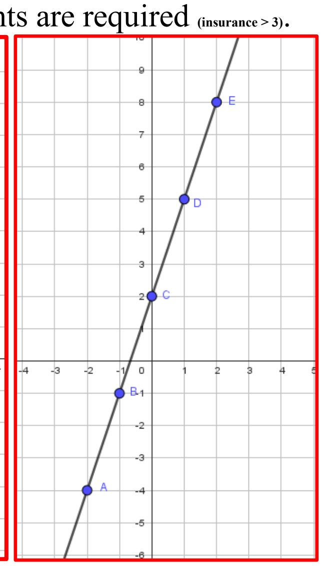
$y = 3x + 2$

1. Solve for y
2. Create the table of values
3. Plot the points on a grid
4. Connect points with a line
5. The successive differences with the x- and y-values can show the **rise** over **run** or <u>**slope**</u>.

run ↓ (-1, -1, -1, -1) rise ↓ (-3, -3, -3, -3)

x	Work Example	y
-2	3(-2) + 2 = -4	-4
-1	3(-1) + 2 = -1	-1
0	3(0) + 2 = 2	2
1	3(1) + 2 = 5	5
2	3(2) + 2 = 8	8





Results is a line with a slope of 3 $\left(\frac{rise}{run} = \frac{-3}{-1} = 3\right)$, and a y-intercept at C(0, 2).

Note the values of slope and y-intercept in the equation!

Practice using tables will/improve the solving linear equations. One needs to be able to determine the equation from a set of points or from the graph.

Slope-Intercept Methods (Interactive Help)
https://www.geogebra.org/m/mEs37yMj#material/hp44vxvk

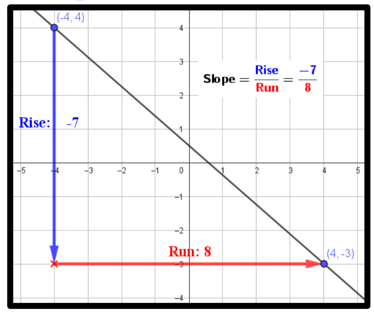
Solving linear equations:

1. Simplify each side of the equation, i.e., gather the like terms on each side.
2. Isolate the y-term. (Add or Subtract either the x-term and/or the constant terms, as needed.)
3. Simplify such that the y-term is positive, and so the y-term is by itself on one side of the equation.
4. Isolate the y-variable. (Multiply or Divide each term of the equation by the coefficient of y.)
5. Simplify each term, the equation should look similar to this: y = m x + b.
6. Use the expression to find the points on the line by choosing an x-value and solving for the y-value.

Linear Function Summary

Formats:

1. **ax + by + c = 0**, where *a*, *b*, and *c* are real numbers. (**General Form**)
2. **Ax + By = C**, where A, B, and C are whole numbers. (**Standard Form**)
3. **y = mx + b**, where m is the slope and b is the y-intercept (0, b) (**Slope-Intercept Form**)
4. **$y - y_1 = m(x - x_1)$**, where m is the slope and (x_1, y_1) is point on the graph (**Point-Slope Form**)



What do you notice about the x and y variables in all the equations above?

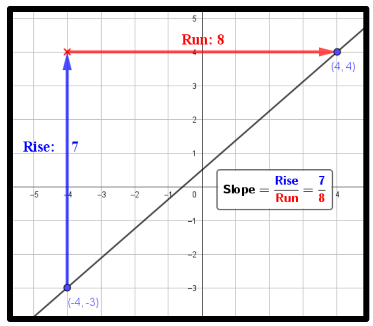
There are **No Exponents** nor **products of variables** in any of them.

What do you notice about the differences between x-values or y-values in a table of values?

The differences of the x-values are constant, **AND** the differences of the y-values are constant. The slope, $\boldsymbol{m} = \frac{y_2 - y_1}{x_2 - x_1}$ *or* $\frac{\textbf{differences of the y-values}}{\textbf{differences of the x-values}}$ *or* $\frac{\textbf{rise}}{\textbf{run}}$, is found this way.

Finding the slope on a graph (example from lower graph):

https://www.geogebra.org/m/mEs37yMj#material/nqeb3fee



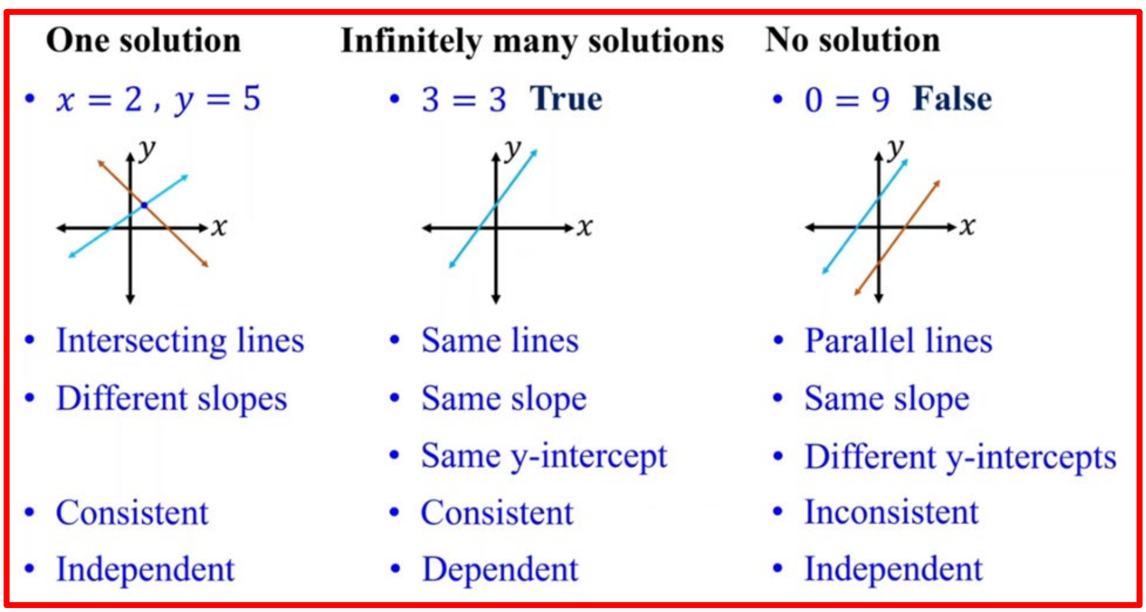
1. Start from the leftmost (or rightmost) point **(-4, -3)**
2. Move vertically (either up or down) from that point to the y-value of the second point **rise = 7**
3. Move horizontally to the right point **(4, 4)**. **Run = 8**
4. The result is the slope of the line. **Rise** : **Run**
5. $m = \frac{\boldsymbol{rise}}{\boldsymbol{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-2)}{4-(-5)} = \frac{7}{9}$.

Systems of Linear Equation

(Recommended video for all students.)

https://www.youtube.com/watch?v=F77xmwmZZsU

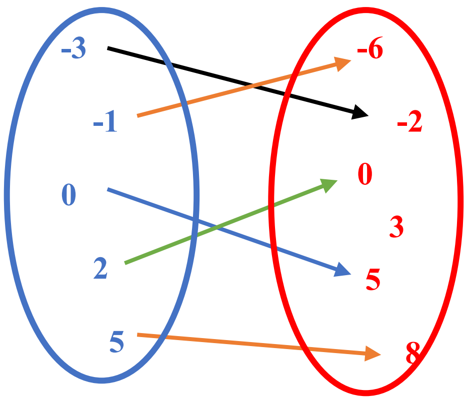
Solving linear equation problems involve **Systems of Linear Equations**. This chart comes from an excellent video discussing these kinds of problems. Several methods are discussed on the understanding of slope-intercept solutions. Many times solving by process of eliminating a variable by adding or subtracting the equations to eliminate one variable is the fastest method. It also may entail multiplying one or both equations by a constant to eliminate a variable by addition or subtraction.



https://www.geogebra.org/m/mEs37yMj#material/G76vE2FF

Domain and Range (x-values and y-values or independent variables and dependent variables)

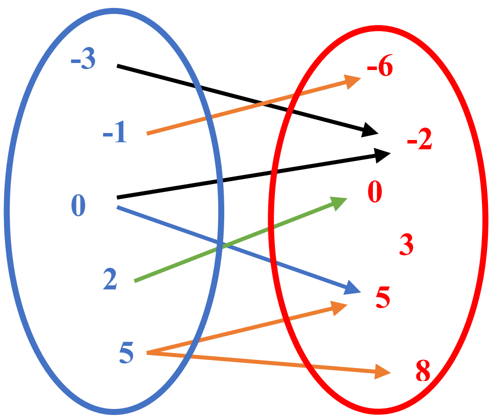
Domain Range



Domain x Independent	Range y Dependent
-3	-2
-1	-6
0	5
2	0
5	5

The set of points at the left is a **function** since the domain values have a **one-to-one** pairing relationship with the domain-to-range elements.

Domain Range



Domain x Independent	Range y Dependent
-3	-2
-1	-6
0	5
0	-2
2	0
5	5
5	8

The set of points at the left is a **relation** since the domain values of 0 and 5 which have two range values.
If we eliminate the 0 and 5 from the domain, the first four ordered pairs would be a function.

All functions are relations, but **NOT** all relations are functions.

Preparation for quadratic equations include a review of the **Distributive Properties of Equality**, a(b + c) = ab + ac or a(b – c) = ab – ac. These properties are expandable to binomial multiplication in the following manner. (**See 07 Polynomial Overview, Sections 10-13 for lessons**.)

(a + b)(c + d)
a(c + d) + b(c + d)
ac + ac + bc + bd

For example,

★Expanding ↓ ↑ **★Factoring**

(2x + 3)(3x – 5)
2x(3x – 5) + 3(3x – 5)
2x•3x – 2x•5 + 3•3x – 3•5
$6x^2 - 10x + 9x - 15$
$6x^2 - x - 15$

This is an example of expanding binomial products. Reversing the procedure is **factoring**.

From the example above a = 6, b = -1, and c = -15 of $ax^2 + bx + c$, so multiply a•c = 90. The factor pairs of 90 are (1,90), (2,45), (3,30), (5,18), (6,15), (9,10). Since b = -1, the only pair with a difference of 1 is (9,10). This means that 9 and -10 are the values that equal -1, yielding the second step of the factoring process. The next step is to reverse distribute the terms to become 2x(3x – 5) + 3(3x – 5), yielding the solution.

Quadratic Equation Summary

Formats:

1. **y = ax² + bx + c**, where *a*, *b*, and *c* are real numbers. (**Standard Form**) GED tested format.
2. **y = a(x – m)(x – n)**, where *a*, *m*, and *n* are real numbers. (**Factored Form**)
3. **y = a(x – h)² + k**, where *a* is a real numbers and (*h, k*) is the maximum or minimum point of the parabola formed. (**Vertex Form**, this is done in the Algebra II.)
4. $\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$, where *a*, *b*, and *c* are the values from the standard form only. This can be used to find all the solutions of any quadratic equation. However, if an express factors, **factoring** is faster and simpler.
5. A function table is a great tool to visualize the points on a graph.

What do you notice about the x and y variables in all the equations above?

One x-variable is squared, or two x-variables are multiplied times each other.

Note: It is not uncommon to see the *y*-variable replaced by something like *f(x)*, referring to the function *f* over the variable *x*.

The standard form of a **Quadratic Equation** **(Kaplan, pp. 360-363)**

$$y = ax^2 + bx + c$$

The coefficients **a**, **b**, and **c** of the quadratic equation are used here.
y is sometimes written as f(x) {function notation}.

Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant: $\boldsymbol{b^2 - 4ac}$ (determines the type of roots)

Example: y = 5x² + 3x – 6; a = 5, b = 3, and c = -6

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 5 \times (-6)}}{2 \times 5}$$

$$x = \frac{-3 \pm \sqrt{9 - 20 \times (-6)}}{10}$$

$$x = \frac{-3 \pm \sqrt{9 + 120}}{10}$$

$$x = \frac{-3 \pm \sqrt{129}}{10}$$

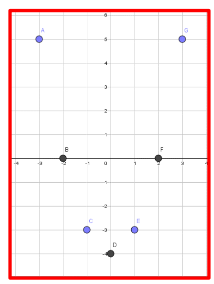
Check your setup work:
https://www.geogebra.org/m/mEs37yMj#material/W93ZvjSN

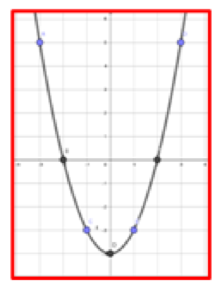
$x^2 - y - 4 = 0$
$y = x^2 - 4$

1. Solve for y
2. Table Values
3. Plot points
(5 min, 7 insure)
4. Connect Pts

x	Work	y
-3	$(-3)^2$-4 = 5	5
-2	$(-2)^2$-4 = 0	0
-1	$(-1)^2$-4 = -3	-3
0	$(0)^2$-4 = -4	-4
1	$(1)^2$-4 = -3	-3
2	$(2)^2$-4 = 0	0
3	$(3)^2$-4 = 5	5

See Table 2 last page.





Factoring Quadratic Expressions:

Factoring when a = 1: $x^2 + 5x + 6$; **a = 1, b = 5, c = 6**

1. Multiply a•c =1•6 = 6 {a•c = m•n and b = m + n}
2. Factor pairs 6 are: **(1,6), (2,3)**; which of these add to b?
3. Since b = 5 is 2 + 3, m_f = 2 and n_f = 3
   - **a.** $\mathbf{m_f + n_f = 2 + 3}$ (the b-value of 5)
   - **b.** $\mathbf{ax^2 + n_f\,x + m_f\,x + c}$
   - **c.** $x^2 + 2x + 3x + 6$
   - **d.** $x(x + 2) + 3(x + 2)$
   - **e.** $(x + 2)(x + 3)$

Alternately,

- b. $\frac{a}{m_f} = \frac{1}{2}$ and $\frac{a}{n_f} = \frac{1}{3}$
- c. $(a\,x + m_f)(a\,x + n_f)$
- d. $(1x + 2)(1x + 3)$ **{1x ≡ x}**

Factoring when a ≠ 1: $5x^2 + 12x + 4$; **a=5, b=12, c=4**

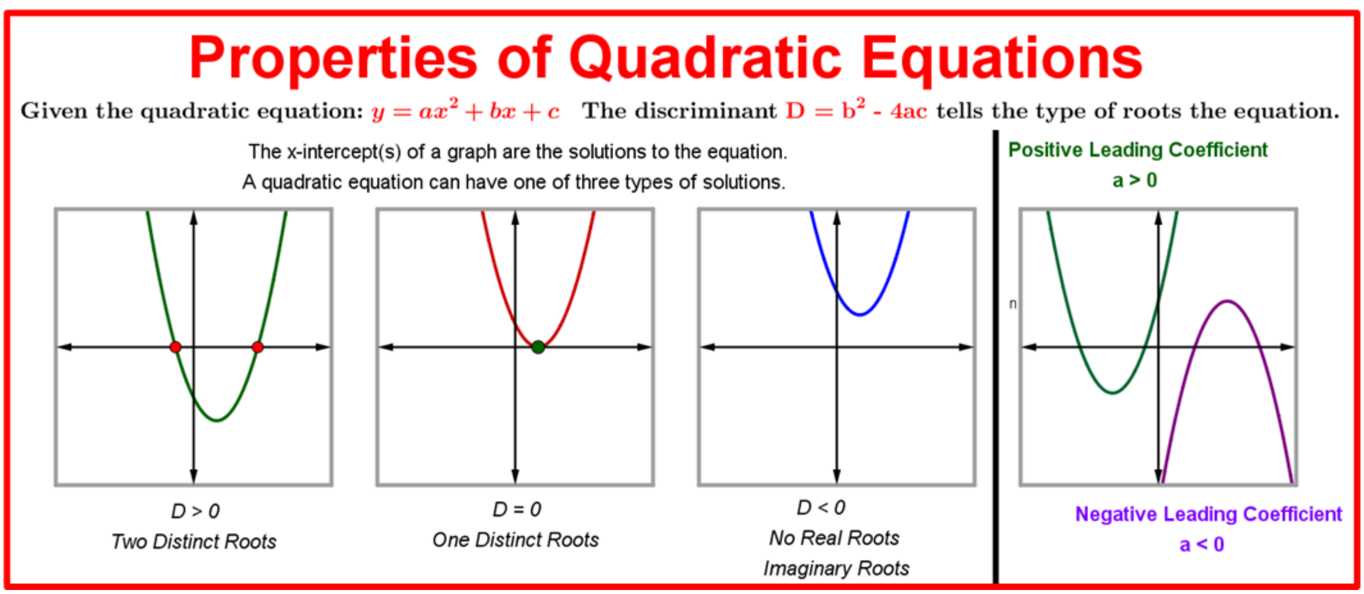
1. Multiply a•c = 5•4 = 20
2. The factor pairs of 20: (1,20), (2,10), (4,5)
3. Since only (2,10) add to b = 12
4. So, m_f = 2 and n_f = 10
   - **a.** $\mathbf{5x^2 + m_f\,x + n_f\,x + 4}$
   - **b.** $5x^2 + 2x + 10x + 4$
   - **c.** $x(5x + 2) + 2(5x + 2)$
   - **d.** $(x + 2)(5x + 2)$

Alternately,

- b. $\frac{a}{m_f} = \frac{5}{10}$ *or* $\frac{1}{2}$ and $\frac{a}{n_f} = \frac{5}{2}$ **{reduce when needed!**
- c. $(a\,x + m_f)(a\,x + n_f)$
- d. $(x + 2)(5x + 2)$ **{1x ≡ x}**

Solving Quadratic equations:

1. Simplify each side of the equation, i.e., gather the like terms on each side.
2. Isolate the y-term if one exists. If no y-term, arrange the expression into this form $ax^2 + bx + c = 0$. If there is a y-term is positive or 0, and so the y-term is by itself on one side of the equation……...????
3. Isolate the y-variable. (Multiply or Divide each term of the equation by the coefficient of y.)
4. Factor the quadratic expression or use the quadratic formula.
5. $a x^2 + b x + c = 0$
   $x^2 + 3x - 10 = 0$, so a = 1, b = 3, c = -10
6. Use the product of a•c to find a set of factor pairs that add to 'b'. {1•(-10) = -10: (1, -10), (2, -5), (5, -2), (10, -1). Since only 5 + -2 = 3, these are used to rewrite the trinomial.
7. Use those factors to rewrite the trinomial into a polynomial with 4 terms.
8. $x^2 + 5x - 2x - 10 = 0$
   x(x + 5) - 2(x + 5) = 0
   The first two terms will factor into a monomial times a binomial **x(x + 5)** and the last two terms will factor into a monomial times a binomial **-2(x + 5)** (the same one).
9. This binomial **(x + 5)** will factor out of the new expression giving the product of the common binomial with another binomial.
10. **(x + 5)(x - 2) = 0** These binomials when multiplied together will equal the original trinomial.
11. Whenever the product of values equals zero, then one of the values must be zero. Therefore, x + 5 = 0 or x - 2 = 0.
12. x + 5 = 0 → x = -5 and x – 2 = 0 → x = 2. So the two points where the graph crosses the x-axis is at the coordinates (-5, 0) and (2, 0) are the solutions of the given quadratic equation.
13. By design quadratic equations have at most two real solutions, but a single real solution is possible. Solutions not studied for the GED® are the situations where there are imaginary (not real) solutions.

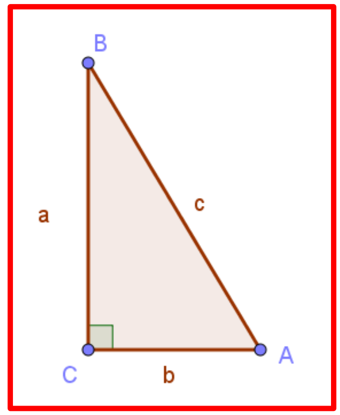


Short videos
Number of x-intercepts: https://www.youtube.com/shorts/WgfNk6h7ONk
Which method to use? https://www.youtube.com/shorts/e7b_LK25IIU

Pythagorean Theorem $a^2 + b^2 = c^2$

a and **b** are perpendicular sides of triangle, **c** is hypotenuse.

This theorem is used for multitudinous basic exam questions wherever perpendicular lines can be found. Be prepared to use it multiple times. At the **College Ready Level** it could one step in solving surface area or volume problems.



Pythagorean Triples in use for 10,000 years

[3, 4, 5] $3^2 + 4^2 = 5^2 \leftrightarrow 9 + 16 = 25$; [5, 12, 13] $5^2 + 12^2 = 13^2 \leftrightarrow 25 + 144 = 169$

$[1, 1, \sqrt{2}]$ is for the 45°, 45°, 90° triangle. $[1, \sqrt{3}, 2]$ is for the 30°, 60°, 90° triangle.

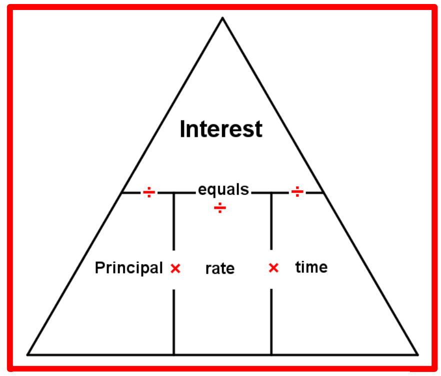
The Pythagorean Theorem can be alone (basic level), or part of another problem (College Ready level).

https://www.geogebra.org/m/mEs37yMj#material/uTvy5sKR Pythagorean Theorem
https://www.geogebra.org/m/mEs37yMj#material/p4GbfQkq With semicircles

simple interest $I = Prt$

(I = interest, P = principal, r = rate, t = time)

This is used to find out how much it costs to borrow money, or how much you can earn when you save money. **(Kaplan, pp. 272-273)**



Cover the variable for which you want a solution, the formula is parts not covered.

$$I = Prt \text{ or } P = \frac{I}{rt} \text{ or } r = \frac{I}{Pt} \text{ or } t = \frac{I}{Pr}$$

Other variations of interest formulae are used for percent change, percent increase, percent decrease, and compound interest which is not tested.

What "percent" of the whole is the part? r% × whole = part

What percent is the part of the whole? $r\% = \frac{r}{100} = \frac{\text{part}}{\text{whole}}$

The percentage of what number is the part? $whole = \frac{part}{percentage}$

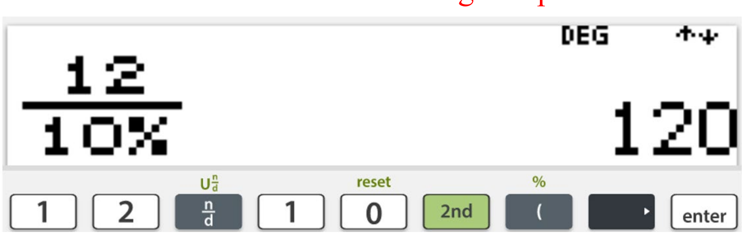
There are three basic types of "percent" problems: (Kaplan, pp. 266-269)

- Find a given "percent" of a given number. For example, find 25% of 640.

- Find a percentage given by two numbers. For example, 15 is what percent of 50?

Student then must change the decimal 0.3 to a percent, 30%.

- Find a number that is a given percent of another number. For example, 10% of what number is 12?



The student must change the percentage of 10% to a decimal, 0.10.

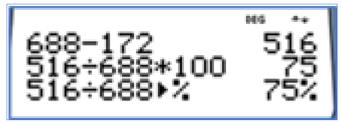
Expect to be asked to find any of the parts of the equations for percentage, distance, or cost.

How do you find the percent of change? **(Kaplan, pp. 274-275)**
First: work out the difference (increase/decrease) between the two numbers you are comparing.
Then: divide the increase/decrease by the original number and multiply the answer by 100.

% increase = Increase ÷ Original Number × 100
or
% decrease = Decrease ÷ Original Number × 100

Calculator screens shown are from a TI30XS Multiview.

Find the percent of change from 688 qt to 172 qt.
{Going from a high value to a lower value is a decrease.}

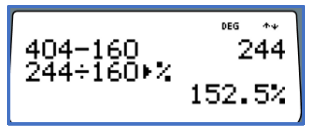


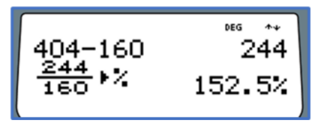
%decrease = decrease ÷ Original Number x 100
Decrease = 688-172=516
516 / 688 * 100 = 75 % (standard calculator)

Shown above are two ways to find the %:
First & Second line, common calculator.
First & Third, TI30XS Multiview function.
A percent can be given by a non-percent value (any ratio format: fraction, mixed number, etc. which can convert to a %)

Find the percent of change from 160 cups to 404 cups.
{Going from a low value to a higher value is an increase.}

% increase = Increase ÷ Original Number × 100
Increase = 404 – 160 = 244
244/160 *100 = 152.5 % (standard calculator)

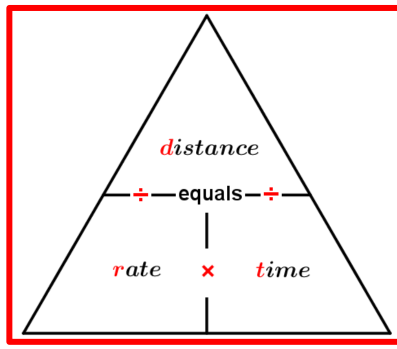




distance formula d = rt

d is distance, ***r*** is rate and ***t*** is time.

$$d = rt \text{ or } r = \frac{d}{t} \text{ or } t = \frac{d}{r}$$



Cover the variable for which you want a solution, the formula is parts not covered.

(Kaplan, pp. 232-233)

total cost total

cost = (number of units) × (price per unit)

This is the cost for buying multiple items of the same price.

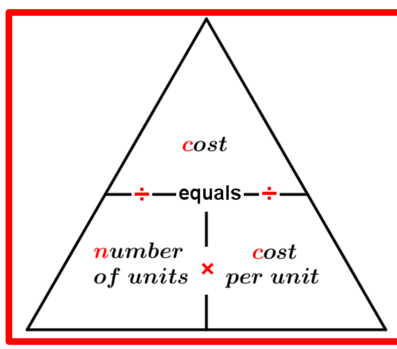
$$cost = (number\ of\ units) \times (price\ per\ unit)$$

Or

$$(number\ of\ units) = \frac{cost}{(price\ per\ unit)}$$

Or

$$(price\ per\ unit) = \frac{cost}{(number\ of\ units)}$$



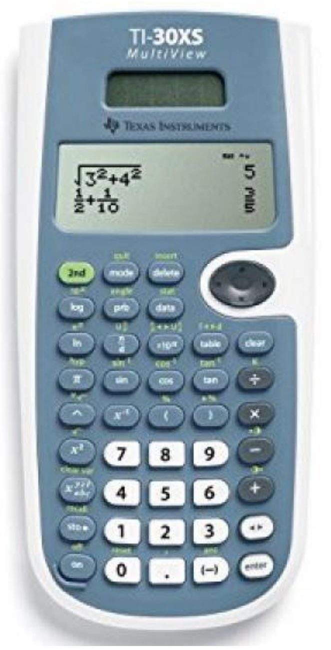
Expect to be asked to find any of the parts of the equation for percentage, distance, or cost.

The formula triangles (rectangles) have been used to assist students who have not learned algebra at a time when students need to be able to use the formula variations needed to solve problems. Also, electricians and other trades use them as tools of their trades.

GED®/HSE test only Middle School Geometry; they do **not** test HS Geometry.

Perimeters and Areas of plane figures: circles, triangles, quadrilaterals **(Kaplan, pp. 388-397)**
Surface Areas and Volumes of solid: spheres, cubes, pyramids, and other solids **(Kaplan, pp. 398-403)**
Students need to learn to adapt the formula for composite plane-figures and solids. **(Kaplan, pp. 404-407)**
Ratio and Proportions are another topic. **(Kaplan, pp. 260-263)**

The ONLY calculator you can use on the GED exam is the TI30 XS Multiview.

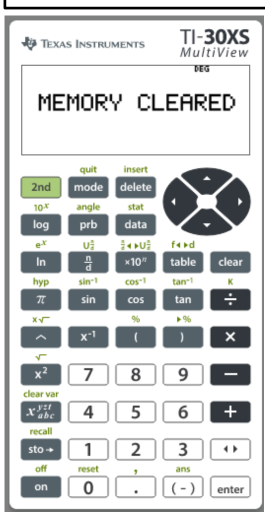


Available from:

- Amazon
- Office Depot
- Target
- Walmart

Cost $20-30

An on-screen version is provided during the GED® Exams for Math, Science, and Social Studies. You may use your personal device when testing at a testing center. For Virtual Home Testing, you must use the provided device.



GED® PROGRAM

CALCULATOR REFERENCE GUIDE

Working with complex problems on the test is simple when you use this guide to understand what order to click the buttons in the on-screen calculator. The GED® test calculator is the TI-30XS.

BASIC ARITHMETIC

To perform basic arithmetic, enter numbers and operation symbols using the standard order of operations.

Example: $8 \times -4 + 7 =$

[8] [×] [(–)] [4] [+] [7] [enter]

The correct answer = **–25**

SCIENTIFIC NOTATION

To perform calculations with scientific notation, use the [×10ⁿ] key.

Example: $7.8 \times 10^8 - 1.5 \times 10^8 =$

[7] [.] [8] [×10ⁿ] [8] [►] [–] [1] [.] [5] [×10ⁿ] [8] [enter]

The correct answer = **630000000**

MIXED NUMBERS

To perform calculations with mixed numbers, use [2nd] [n/d]. As with fractions, the answer will automatically be formatted in reduced form.

Example: $12\frac{5}{6} - 1\frac{1}{2} =$

[1] [2] [2nd] [n/d] [5] [▼] [6] [►] [–] [1] [2nd] [n/d] [1] [▼] [2] [enter]

The correct answer = $\frac{34}{3}$

FRACTIONS

To perform calculations with fractions, use the [n/d] key. The answer will automatically be formatted in reduced form.

Example: $\frac{2}{9} \times \frac{3}{7} =$

[n/d] [2] [▼] [9] [►] [×] [n/d] [3] [▼] [7] [enter]

The correct answer = $\frac{2}{21}$

PERCENTAGES

To calculate with percentages, enter the number, then [2nd] [(].

Example: $40\% \times 560 =$

[4] [0] [2nd] [(] [×] [5] [6] [0] [enter]

The correct answer = **224**

POWERS AND ROOTS

To perform calculations with powers and roots, you will use the following keys:

[x²] [^] [2nd] [x²] [2nd] [^]

Example: $1.2^2 =$

[1] [.] [2] [x²] [enter]

The correct answer = **1.44**

Example: $7^4 =$

[7] [^] [4] [enter]

The correct answer = **2401**

Example: $\sqrt{529} =$

[2nd] [x²] [5] [2] [9] [enter]

The correct answer = **23**

Example: $\sqrt[3]{1728} =$

[3] [2nd] [^] [1] [7] [2] [8] [enter]

The correct answer = **12**

TOGGLE KEY

The answer toggle key [◄►] can be used to toggle the display result between fraction and decimal answers, exact square root and decimal, and exact pi and decimal.

Example: $\frac{9}{10} =$

[n/d] [9] [▼] [1] [0] [enter] [◄►]

The correct answer = **0.9**

This calculator reference sheet is provided for most items on the GED® test — Mathematical Reasoning, as well as certain items on the Science and Social Studies tests.

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https://ged.com/wp-content/uploads/calculator_reference_guide.pdf

Geometric Figures and Definitions

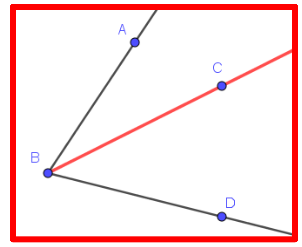


Figure 1

- **Adjacent angles→**two angles that share a common ray; ∡ABC is adjacent to ∡CBD, in Figure 1.
- **Angle→**two rays having the same vertex point, **B**, and more than a 0° arc between them in Figure 1: ∡ABC, ∡CBD, ∡ABD
- **Complementary angles→**two angles whose sum is 90°; if they are adjacent angles, they form a right angle. Figure 4. https://www.geogebra.org/m/mEs37yMj#material/G9De98Xv
- **Line→**has no thickness and extends endlessly in both directions
- **Intersecting lines→**have different slopes; line *h* intersects lines *g* and *f*, can intersect at any angle. Figure 1 shows ⊥ lines intersecting.
- **Parallel lines→**two or more lines that have the same slope; lines *f* and *g* are parallel→ *f*||*g*
- **Perpendicular lines→**two lines that intersect at a 90° or right angle; two lines whose slopes have a product of -1. These definitions will assist in slope problems. Line *h* is perpendicular to lines *f* and *g*. $h \perp f$ and $h \perp g$ and $\frac{-2}{1} \times \frac{1}{2} = -1$.
- **Point→**a location in space; **in Figure 1: points A, B, C, D**
- **Ray→**a directed line from one point through another point, but never ending

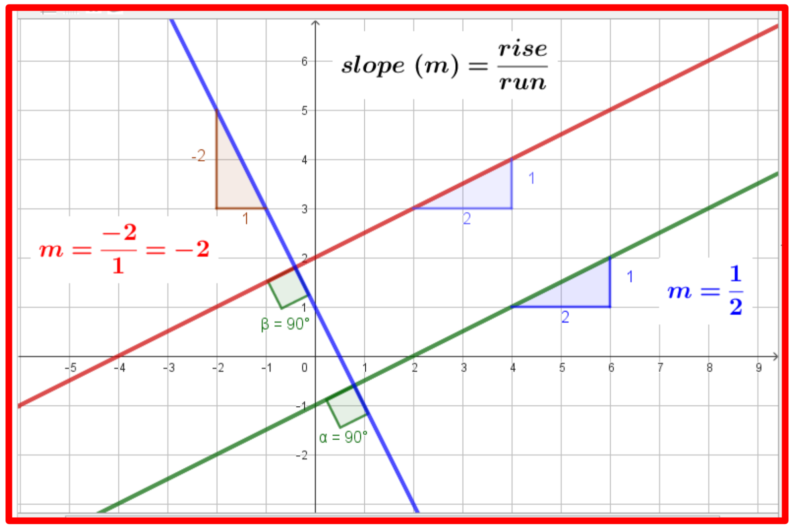


Figure 2

- **Right angle→**an angle whose measure is 90°; see ⊾ in Figure 2 above.
- **Regular polygons→**all sides are congruent, and all angles are congruent. On HSE exams, all plane figures with more than 4 sides where a formula is use are usually regular.
- **Segment→**the line between two distinct points; link between two points
- **Supplementary angles→**two angles whose sum is 180°; if they are adjacent angles, the form a straight line. Figure 4. https://www.geogebra.org/m/mEs37yMj#material/hXYrYnCD

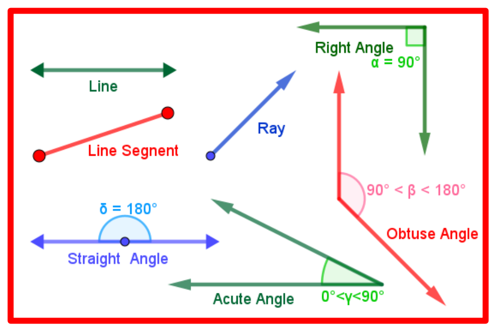


Figure 3
Naming Geometric Objects
https://www.geogebra.org/m/nhebw3qp

Coordinate System Basics:

https://www.geogebra.org/m/mEs37yMj#material/smmguthf

The Sum of the Interior Angles of a Triangle:

https://www.geogebra.org/m/mEs37yMj#material/VgEe9K5d

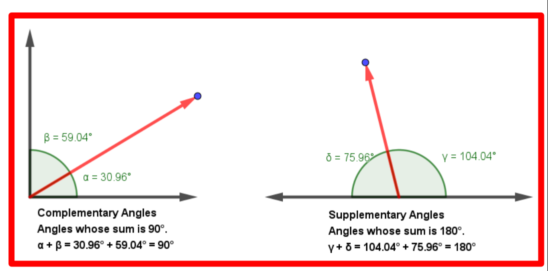
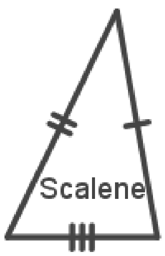


Figure 4
https://www.geogebra.org/m/mEs37yMj#material/jnwbyvds

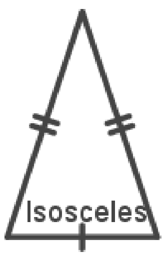
Triangle Properties

All triangles are described using both descriptive information, once by the length of their sides and once by the size of their angles. Examples of commonly discussed vocabulary used in word problems:

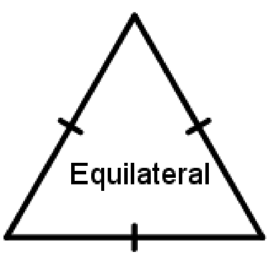
Side Properties



All Sides Different



Two Sides the Equal



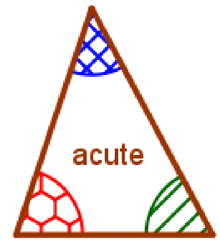
All Sides Equal

- **The sum of all a triangle's angles is always 180°.** https://www.geogebra.org/m/mEs37yMj#material/VgEe9K5d
- The angles of a scalene triangle can be acute, right ⊾, or obtuse.
- The angles of an isosceles triangle can be all acute, one right ⊾ and two acute, or one obtuse and two acute.
- The angles of an equilateral triangle can only be acute, and all angles are 60°.
- The sides of an acute triangle can be scalene, isosceles, or equilateral, never right ⊾ triangle.
- The sides of the right triangle can be either scalene or isosceles.
- The sides of an obtuse triangle can be scalene or isosceles, never right ⊾ triangle.
- All equilateral triangles are equiangular triangles, each angle 60°. Equilateral ↔ Equiangular ↔ 60° angles and ≅ sides.
- Within an isosceles triangle, the angles opposite the congruent sides, ≅ sides, are congruent angles, ≅ angles.
- **The sum of any two sides of a triangle must be greater than the third side.** https://www.geogebra.org/m/mEs37yMj#material/EH27snzx

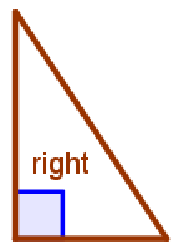
$$\text{side}_a + \text{side}_b > \text{side}_c$$

Triangle Properties
https://www.geogebra.org/m/mEs37yMj#material/nndygtmt

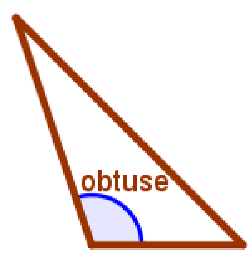
Angle Properties



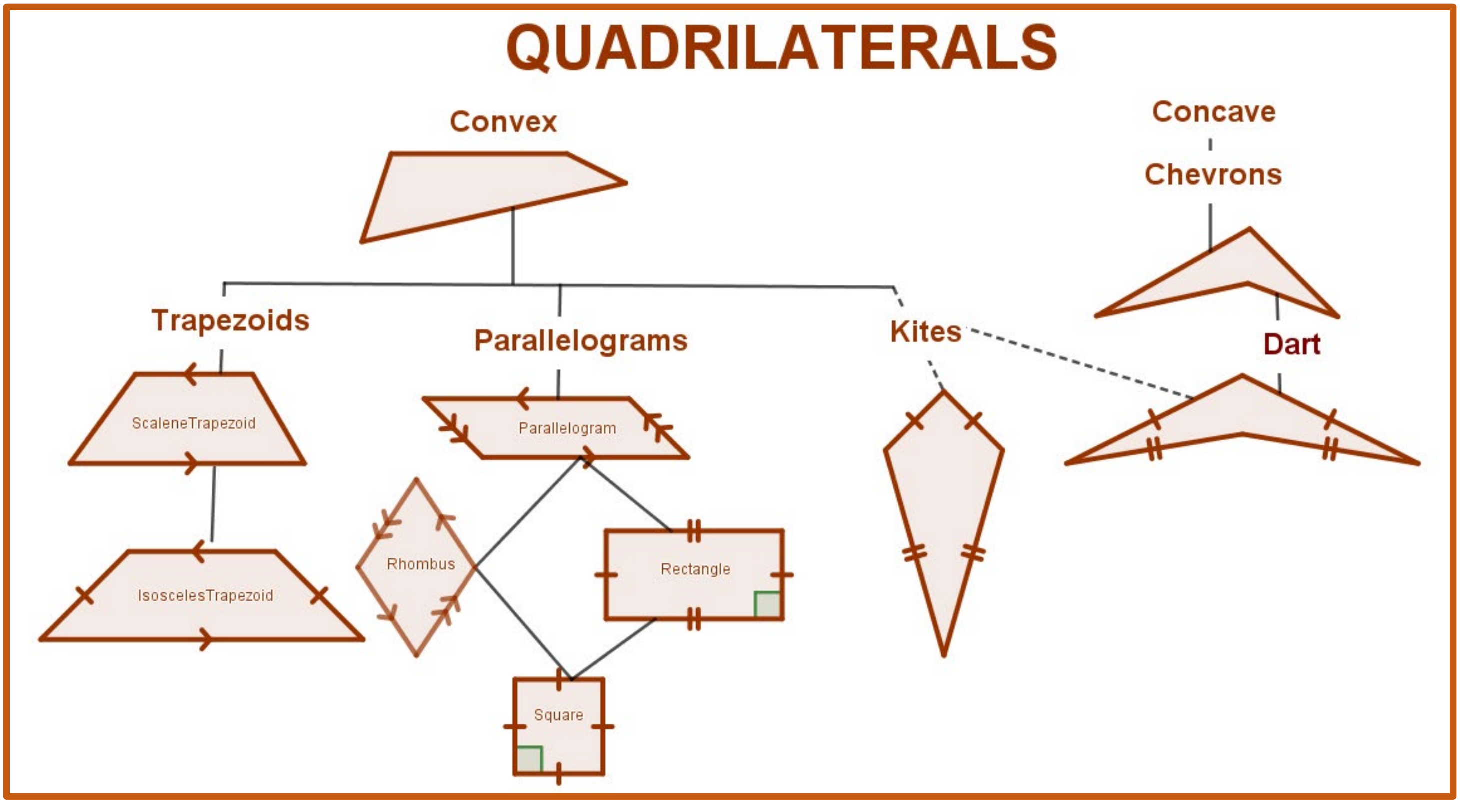
All Angles are < 90°



One Angle is = 90°



One Angle is > 90°



GED®/HSE test currently only tests the named trapezoids and the parallelograms shown above.

Trapezoids have one set of parallel lines. {either isosceles or scalene}

Parallelograms have two sets of parallel lines on opposite sides.

Parallelogram: opposite sides parallel || and congruent ≅

Rectangle: opposite sides congruent, all angles are right ∟ angles.

Square: all sides congruent, right ∟ angles

Rhombus: all sides congruent, no right ∟ angles (use the formulas for a parallelogram)

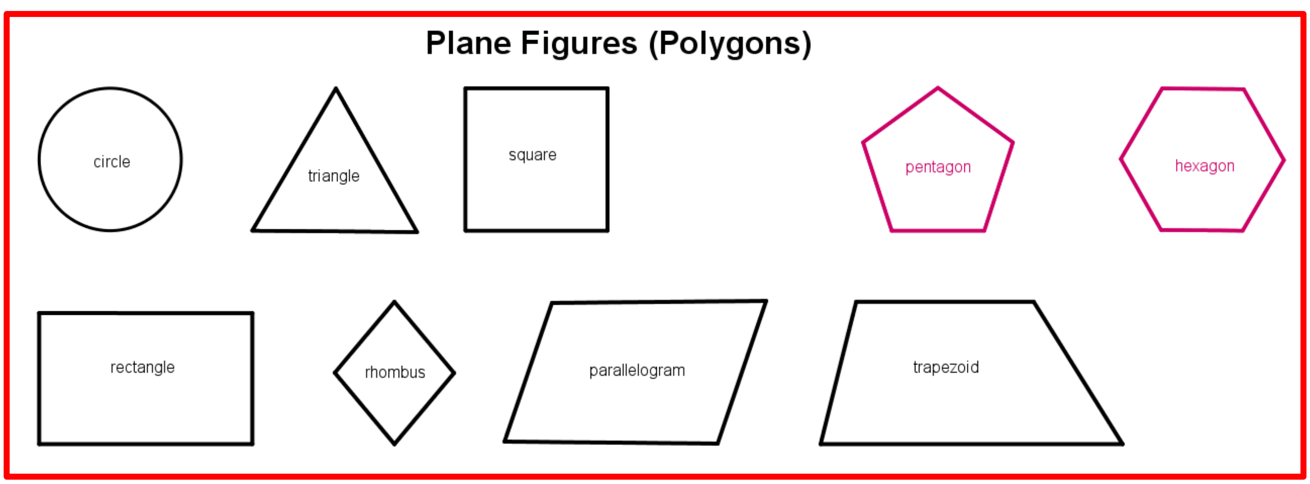
Kites (Darts) have adjacent congruent sides and one pair of congruent angles making them fit the definition of a kite. The traditional kite shape is convex; however, the dart is concave.

Chevrons have no parallel lines.

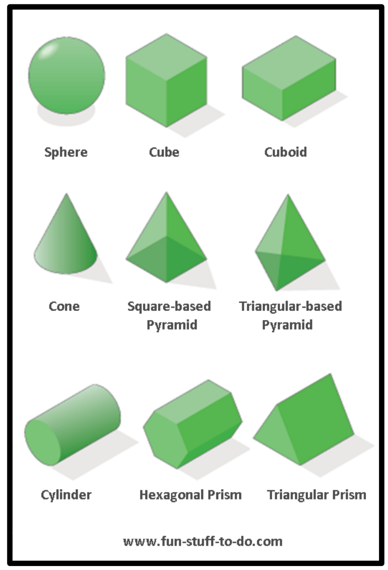
Convex quadrilaterals diagonals are in the interior of the quadrilateral.

Concave quadrilaterals one diagonal is in the exterior of the quadrilateral.

Reddish shape's formulae are not tested but may be mentioned.



Geometric Solids



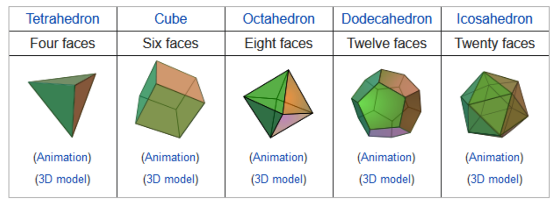
Geometric Solids can be seen in daily use:

1. Sphere: various balls in multiple sports
2. Cube: sugar, boxes, electronics containers
3. Cuboid: Kleenex boxes, rectangular prisms
4. Cone: ice cream, snow cones, water cups
5. Cylinder: cans, paper towel rolls, pipes
6. Square Pyramids: Egyptian, Aztec, Mayans, etc.
7. Prisms: jewelry, scientific tools, etc.
8. Tetrahedron is an equilateral triangular based pyramid.
9. Can you think of other examples?

    a. ______________________________________

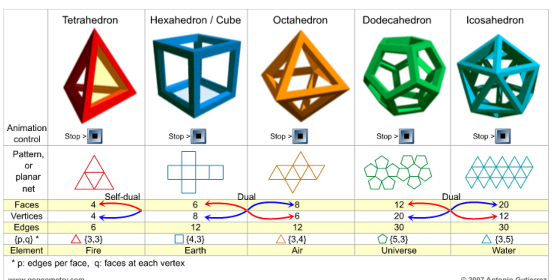
    b. ______________________________________

Example site for interactive Solids or 3D Shapes
https://www.geogebra.org/t/solids?lang=en

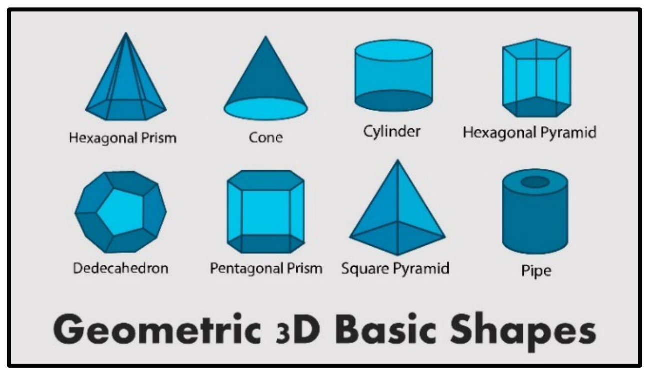


Platonic Solids (Theaetetus' Theorem) Click on these links to view them in 3D. https://en.wikipedia.org/wiki/Platonic_solid

These shapes are the Dungeons & Dragons dice.



http://www.gogeometry.com/solid/platonic-solids-html5-animation-ipad-nexus-7.htm



https://myfreeslides.com/3d-geometric-shapes-for-google-slides-powerpoint/

Commutative (to commute means moving to another position)

$3 + 5 = 5 + 3$

$3 \times 5 = 5 \times 3$

Associative (to associate or someone who is you friend for going to church vs one who is friend for having fun at a park)

$3 + (5 + 7) = (3 + 5) + 7$

$3 \times (5 \times 7) = (3 \times 5) \times 7$

Subtraction and Division do NOT follow the rules of commutativity and associativity above; we need to change all subtraction and division as follows:

Inverse Applications (change in what I do, opposite operation)

$a - b = a + (-b)$… Example: $8 - 3 = 8 + (-3)$

$a \div b = a \times \frac{1}{b}$…. Example: $8 \div 4 = 8 \times \frac{1}{4}$

Distributive (to distribute or to pass out to someone; this allow you to share addition with multiplication)

$a(b + c) = ab + ac$ or $a(b - c) = ab - ac$ or $\frac{b+c}{a} = \frac{b}{a} + \frac{c}{a}$, plus could be a subtration.

Properties of Mathematical Sentences

"**Arithmetic is the Queen of Mathematics.**" Knowledge of the basic properties of math is needed to understand and do mathematics. The following chart summarizes the Field Properties and Properties of Equality and Inequality. These properties apply to all real numbers: *a*, *b*, and *c*. They are used in solving math problems.
{The underlined properties are central to balancing Algebraic operations, resulting in a **solution**.}

Field Properties

Property Adjective Form (Verb Form)	**Addition**	**Multiplication**
Closure	a + b is a real number	ab is a real number
Associative **(associate)**	$(a+b)+c = a+(b+c)$ (4 + 7) + 3 = 4 + (7 + 3)	$(ab)c = a(bc)$ (7•8)5 = 7(8•5)
Commutative **(commute)**	$a+b=b+a$ 5 + 3 = 3 + 5	$ab = ba$ 4•8 = 8•4
Identity **(identify) ID**	$a+0=a=0+a$	$a \cdot 1 = a = 1 \cdot a$
Inverse Addition/Multiplication yield ID values.	$a+(-a)=0=(-a)+a$	$a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a, if\ a \neq 0$
Inverse Applications[¥] **Modify – and ÷ to allow use of the Associative and the Commutative Properties.**	$a - b = a + (-b)$ Directed Number Addition (– becomes +)	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ Reciprocal Multiplication for Division
Distributive **(distribute)**	$a(b \pm c) = ab \pm ac$ and $ab \pm ac = a(b \pm c)$ 5(x + 3) = 5x + 15 by multiplying both x and 3 by 5. This is multiplying across add/subtraction; the reverse is called factoring.	

Properties of Equality and Inequality https://calcworkshop.com/reasoning-proof/properties-equality/

Property		**Equality**	**Inequality**
Multiplicative Property of Zero		$a \cdot 0 = 0 = 0 \cdot a$ Any value times 0 is 0.	
Reflexive	These define an equivalence relation.	$a = a$ (any quantity equals itself)	
Symmetric		If $a = b$, then $b = a$.(Any equation is reversable.)	
Zero Product		If $ab = 0$, then $a = 0$ or $b = 0$.	
Transitive	These properties allow you to balance equations or inequalities involving real numbers. The result is called the **solution**.	If $a = b$ and $b = c$, then $a = c$. If 4 + 2 = 7 – 1 and 7 – 1 = 6, then 4 + 2 = 6.	If a > b and b > c, then a > c. If a < b and b < c, then a < c.
Addition Add same value to each side.		If $a = b$, then $a + c = b + c$. If x – 3 = 8, then x = 11 by adding 3 to both sides.	If $a < b$, then $a + c < b + c$. If $a > b$, then $a + c > b + c$.
Subtraction Sub same value to each side.		If $a = b$, then $a - c = b - c$. If x + 5 = 12, then x = 7 by subtracting 5 from both sides.	If $a < b$, then $a - c < b - c$. If $a > b$, then $a - c > b - c$.
Multiplication Multiply each side by the same value.		If $a = b$, then $ac = bc$. If $\frac{x}{5} = 4$, then x = 2 by multiplying 5 on both sides.	If $a < b$ and $c > 0$, then $ac < bc$. **If $a < b$ and $c < 0$, then $ac > bc$.** If $a > b$ and $c > 0$, then $ac > bc$. **If $a > b$ and $c < 0$, then $ac < bc$.**
Division Divide each side by the same value.		If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$. If 3x = 9, then x = 3 by dividing both sides by 3.	If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$. **If $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$.** If $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$. **If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.**
Substitution		If $a = b$, then b can be substituted for a in any equation or inequality. If 4x + 7y = 11 and x = y, 4x + 7x = 11 or 4y + 7y = 11 by the substitution property.	

Most common errors made by students are with sign number arithmetic and inequality operations using signed numbers.

Do on the left what you do on the right. Do on the right what you do on the left. Extendable to other operations in later work.

Order of Operations

The **Order of Operations** is critical to finding the solution to all math problems. It is a guideline tested on all parts of HSE exams. **PEMDAS** has been used by many in the US! **GERMDAS** and **GEMA** are some of the variations of guide tool. However, evidence has shown that many students make serious errors as they forget how the MD and AS parts of the rule are implemented. The following table shows examples of the operations.

Shelt Algebra: 1.1 Intro to G.E.R.M.D.A.S. https://www.youtube.com/watch?v=E8FQIR6CyDk

Operation	**Example**
Grouping **P**arenthesis ( ), **B**rackets { }, **B**races [ ] Implicit grouping: fraction bars, radical symbols, absolute values, exponent expressions	$4 \times (3+8) = 4 \times 11$; $\lvert 6-11 \rvert$ $\frac{4+8}{12-4} + \sqrt[3]{25+100} = 1+5$ $7 - 15 \div 3 \times 8 + 12$; 5^{3+8-2}
Exponents and **R**oots (binary operations) **E**xponents	$3 \times 5^2 = 3 \times 25$ $\sqrt{9} \times 5^2 = 3 \times 25$
Multiplication and **D**ivision (binary operations) **M**ultiplication and **D**ivision are done in the order the come, left to right.	$4 + 3 \times 5 = 4 + 15$ $12 \div 3 \times 3 + 20 \div 5 \times 2$ $4 \times 3 + 4 \times 2$ $12 + 8 = 20$
Addition and **S**ubtraction (binary operations) **A**ddition and **S**ubtraction are done in the order the come, left to right.	$10 + 3 \times 2 - 8 = 10 + 6 - 8$ $= 16 - 8 = 8$ $8 - 4 \div 2 + 6 = 8 - 2 + 6$ $= 6 + 6 = 12$

Note: There are many alternate acronyms addressing the **Order of Operations**. The acronyms (abbreviations) PEMDAS/BOMDAS, BEDMAS (BIDMAS/BODMAS) are used by many; others criticize their use. When not well understood or recalled, critical errors occur. Other acronym forms of include PEDMAS, PEDMSA, PEMDSA, GEMDAS, GERMDAS, PEMA, BIDMAS, BODMAS, GEMA*, or GEMS. Experience and research have shown many students have problems using these acronyms years later. Memory research has shown the addition of items such as parenthesis or color coding may assist users to remember the proper sequences of operations: **PE(MD)(AS)**, **G(ER)(MD)(AS)** {**BI(DM)(AS)**, **BO(DM)(AS)**}. Here the parenthesis show that values of this type must be completed first when grouped together **in the order they are written in the expression**. Most of the problems associated with acronyms by a learner are they fail to fully learn the order of operations in the first place. Not learning and understanding the concept yields incorrect solutions.

1. **Find each Grouping Symbol, simplify the innermost group first, recall each group may have subgroups where the acronym needs to be followed.**
2. **Once a Grouping is found, simplify exponentials/radicals in the order they occur.**
3. **Then simplify multiplications/divisions in the order they occur.**
4. **Finally, simplify additions/subtractions in the order they occur. Then return to Step 1 for the next Grouping level.**
5. **Once all Groupings are done, repeating Steps 2-5, as needed, will finalize the process.**
6. **While doing the Order of Operations, you will go through each part multiple times.**

Legend
Parenthesis
Grouping
Brackets/Braces

Exponents
Indices
Orders

Multiplication
Division

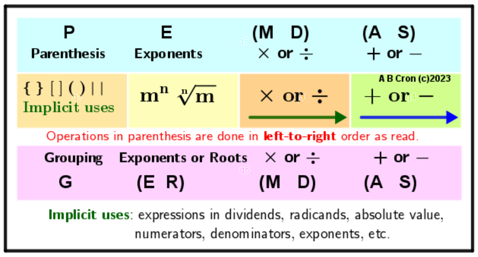
Addition
Subtraction

Four Rules for Order of Operations

1. Perform calculations inside **Grouping Symbols** and Implicit Groupings first.
2. Solve **Exponents and Roots**
3. Perform all **Multiplication and Division** in order from left to right.
4. Perform all **Addition and Subtraction** in order from left to right.
5. Positive/Negative conditionals

*GEMA, see **Nix the Tricks** Chapter 2, Section 8 (Free download, http://nixthetricks.com/.)

While Commutative Properties applies ONLY to addition and multiplication. They do NOT pertain to subtraction or division. However, mathematicians rewrite subtraction and division problems in their additive or multiplicative inverse forms: $\mathbf{a - b = a + (-b)}$ and $\mathbf{\frac{a}{b} = a \div b = a \cdot \frac{1}{b},\ b \neq 0}$. This allows students to reassign the operations using – and ÷ into a + and × formats which are commutative. This allows mathematicians to apply the Commutative and Associative Properties to their problems eliminating division or subtraction operations. Reciprocals: $a \times \frac{1}{a} = 1$. **Perpendicular lines** have slopes which are negative reciprocals of each other: $a \times -\frac{1}{a} = -1$.



The major groups are Grouping, Exp/Roots, Mult/Div, Add/Sub (GEMA).
The process is do all operations within groups following EMA.
When all groups are gone perform EMA on remainder.
The sign of a signed number has the lowest precedence.

All mentioned acronyms fit into the parts if this chart.

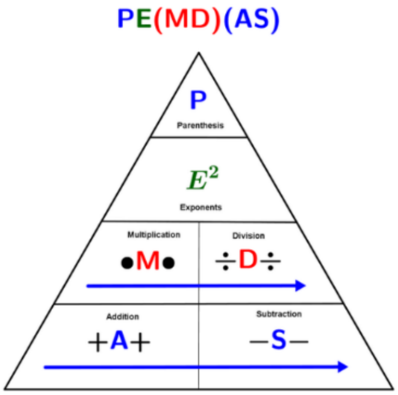
PE(MD)(AS) [Simplify expressions] and (SA)(DM)EP [Solving equations]
G(ER)(MD)(AS) [Simplify expressions] and (SA)(DM)(RE)G [Solving equations]

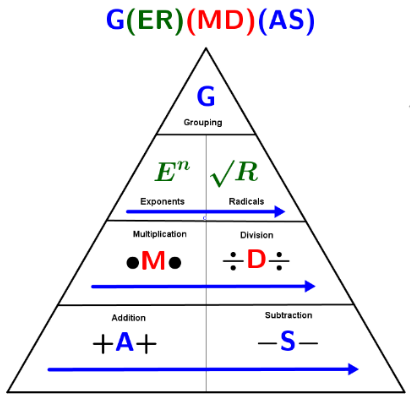
Any Order of Operations acronym can be used in the chart at the left.

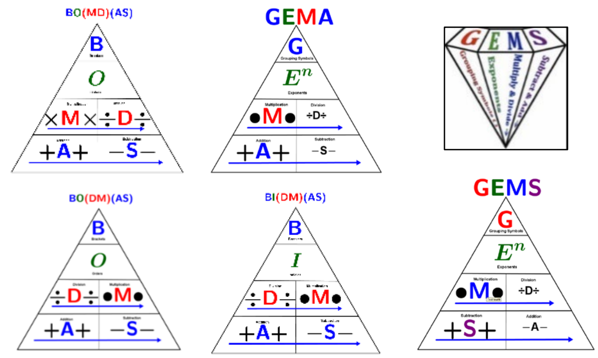
The seven triangles below are representations of common Order of Operation concepts based on a concept introduced by Jerry Ameis.

Ameis, Jerry A. (2011). The Truth About PEDMAS. NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS. 414-420. Volume 16: Issue 7.

The Ameis-Cron Triangle representations of Order of Operations Guidelines:







Exponents/radicals, multiplication/division, and addition/subtraction are **binary** operators requiring two values.
A **function** operation has no operators! **Absolute Value** expression which may include implicit groupings.
As **unary operators**, the precedence of **Positive** & **Negative** numbers have the lowest priority in the hierarchy of the Order of Operation.

Simplify each of the following:

1. $7 - 24 \div 8 \times 4 + 6$
2. $18 \div 3 - 7 + 2 \times 15 \div 6$
3. $6 \times 4 \div 12 + 72 \div 8 - 9$
4. $-2(1 \times 4 - 2 \div 2) + (6 + 2 - 3)$
5. $-3[(3 - 4 \times 7) \div 5] - 2 \times 24 \div 6$
6. $(1^4 \times 3^2 + 4^3) - 2^5 \div 4$
7. $(12 \div 3 \times 2 - 2 \times 5)^2 + (4 - 8 \div 8 \times 3 + 1)^2$
8. $[(64 \div 4^2 - 2) \cdot 5^3] + 6 \cdot 7 - 7^3$
9. $10 \times 6 - (3 \times (5^2 \div 5) \div 3 \div \frac{1}{3}) + \frac{5^2+5}{9-6}$
10. $-12 \div (27 \div 3^2 \times 7 \div 7) \times 8 - \sqrt{16 - 7}$

1) 1 2) 4 3) 2 4) -1 5) 7 6) 65 7) 8 8) -51 9) 55 10) -35

Basic Geometry Terms Videos

Shelt Algebra: 1.1 Intro to G.E.R.M.D.A.S **https://www.youtube.com/watch?v=E8FQIR6CyDk**

Math Antics - Angle Basics https://www.youtube.com/watch?v=DGKwdHMiqCg
Math Antics - Angles & Degrees https://www.youtube.com/watch?v=_n3KZR1DSEo
Math Antics – Perimeter https://www.youtube.com/watch?v=AAY1bsazcgM
Math Antics – Area https://www.youtube.com/watch?v=xCdxURXMdFY
Math Antics – Volume https://www.youtube.com/watch?v=qJwecTgce6c
Math Antics - Circles, Circumference And Area https://www.youtube.com/watch?v=O-cawByg2aA
Math Antics - Circles, What Is PI? https://www.youtube.com/watch?v=cC0fZ_lkFpQ
Math Antics – Triangles https://www.youtube.com/watch?v=mLeNaZcy-hE
Math Antics – The Pythagorean Theorem https://www.youtube.com/watch?v=WqhlG3Vakw8
Math Antics – Quadrilaterals https://www.youtube.com/watch?v=yiREqzDsMP8
How to use a Protractor to Measure Angles! https://www.youtube.com/watch?v=LPc0imoebzI
https://www.geogebra.org/m/j4UyPdKW#material/tnp9hxsd

Other videos

GeoGebra GED sites: https://www.geogebra.org/m/j4UyPdKW Book 1 links to Book 2 and more
Real Numbers Venn Diagram: https://www.geogebra.org/m/j4UyPdKW#material/Vv5cQRBB
Bell Curves article: https://teacherhead.com/2013/07/17/assessment-standards-and-the-bell-curve/
Understanding Permutations-Combinations: https://www.youtube.com/watch?v=S_f8mQdo3bM
Using the Quadratic Formula: https://www.geogebra.org/m/mEs37yMj#material/W93ZvjSN
Coordinate System Basics: https://www.geogebra.org/m/mEs37yMj#material/smmguthf
Sum of the Interior Angles of a Triangle: https://www.geogebra.org/m/mEs37yMj#material/VgEe9K5d
Example site for interactive Solids or 3D Shapes https://www.geogebra.org/t/solids?lang=en

$\frac{1}{0}$ is undefined since there are no quantity of 0s which will multiply to 1, and any value times $0 \neq 1$. ?•0=1

$\frac{0}{0}$ is undefined since any value times 0 is 0, hence we cannot find a value where this is not true.

Determining the Degree Nature of a Function from successive differences of y-values using its function table.

Table 1: Linear
$y = 2x + 2$

x	y
-3	-4
-2	-2
-1	0
0	2
1	4
2	6
3	8
4	10

Differences of x: -1, -1, -1, -1, -1, -1, -1
Differences of y: -2, -2, -2, -2, -2, -2, -2

Table 2: Quadratic
$y = 3x^2$

x	y
-2	12
-1	3
0	0
1	3
2	12
3	27

Differences of x: -1, -1, -1, -1, -1
First differences of y: 9, 3, -3, -9, -15
Second differences: 6, 6, 6, 6

Table 3: Cubic
$y = 2x^3 - 7$

x	y
-2	-31
-1	-10
0	-7
1	-4
2	17
3	74

Differences of x: -1, -1, -1, -1, -1
First differences of y: -21, -3, -3, -21, -57
Second differences: -18, 0, 18, 36
Third differences: -18, -18, -18