

## Geogebra

Set up the **View** with **Algebra**, **Spreadsheet**, and **Input Bar** (include **Graphics** too if desired).

Enter functions  $f(x)$ ,  $g(x)$ , etc. in the Input Bar.

Enter  $x$ -values in column A of the spreadsheet. Use **Fill Down** to create a sequence of values. (Highlight two cells which define a first value and an increment, then drag the box to the target cells.)

The screenshot shows the Geogebra interface. On the left, the Algebra view displays two functions:  $f(x) = 2x$  and  $g(x) = 2^x$ . On the right, the Spreadsheet view shows a table with columns A, B, and C. Column A contains the values 1, 2, 3, 4, 5. Cell B1 contains the formula  $f(A1)$ . The Input bar is empty.

In cell B1 of the spreadsheet, enter  $f(A1)$ . Highlight this cell and **Fill Down** to fill the remaining desired cells in column B.

Repeat: in cell C1, enter  $g(A1)$  and fill down.

### Dynamic Tables option 1

To view the  $y$ -values one at a time, **Fill Down** one at a time.

### Dynamic Tables option 2

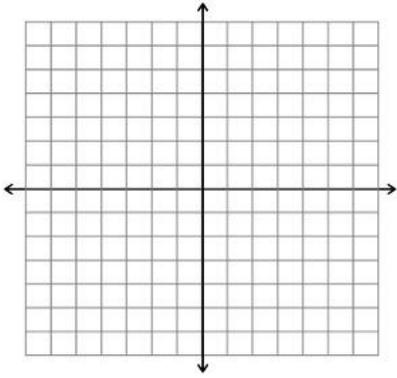
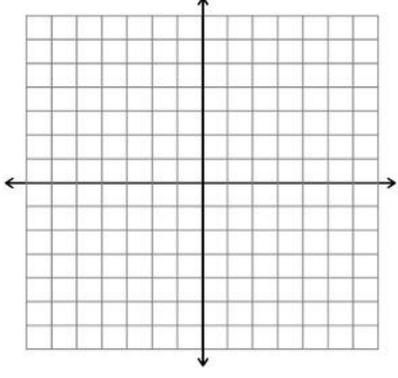
To enter  $x$ -values one at a time, enter an  $x$ -value. Highlight both cells for the  $y$ -values, and **Fill Down** together.

Notes:

- To use row 1 as a “header” row, type “ $X$ ” and “ $f(x)$ ” with quotations in cells A1, B1, etc.
- To guarantee a fraction output, enter this command in the top cell: **`fractiontext(f(A1))`**
- Geogebra calculates in radians, so for  $f(x)=\sin(x)$  and other trig functions, enter this in the top cell: **`f(A1*π/180)`**. The pi symbol is available on the Keyboard.

A blank GeoGebra applet for Table Techniques is at <https://ggbm.at/uw5bqe6g>

### Growth Investigation: Linear Vs. Exponential

| <b>1</b> | <p>Graph these functions:<br/> <math>F(x) = 2x</math><br/> <math>G(x) = 2^x</math></p> <p>What do you observe?</p>   |    |   |           |            |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|----------|--|---|---|-----------|------------|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|
| <b>2</b> | <p>Look at the Table for these functions. Move <u>down</u> the column for each, <b>FILLING DOWN</b> for the value.</p> <p>How does the Y-value change for <math>F(x) = 2x</math>?</p> <p>How does the Y-value change for <math>G(x) = 2^x</math>?</p> <p>At what point(s) do the graphs intersect?</p>                       | <table border="1" style="margin: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">X</th> <th style="padding: 5px;"><math>F(x)=2x</math></th> <th style="padding: 5px;"><math>G(x)=2^x</math></th> </tr> </thead> <tbody> <tr><td> </td><td> </td><td> </td></tr> </tbody> </table> | X | $F(x)=2x$ | $G(x)=2^x$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| X        | $F(x)=2x$  | $G(x)=2^x$  |   |           |            |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|          |  |   |   |           |            |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|          |  |   |   |           |            |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|          |  |   |   |           |            |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|          |  |   |   |           |            |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|          |  |   |   |           |            |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|          |  |   |   |           |            |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|          |  |   |   |           |            |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|          |  |   |   |           |            |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|          |  |   |   |           |            |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| <b>3</b> | <p>Graph these and examine the Table:</p> <p style="text-align: center;"><math>F(x) = 2+3x</math><br/> <math>G(x) = 2 \cdot 3^x</math></p> <p>How are these similar to each other?</p> <p>How does the Y-value change for each?</p> <p>Which equation grows faster?</p>  | <p>Sketch the graphs:</p>    |   |           |            |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| <b>4</b> | <p>What does the 2 in the first equation have in common with the 2 in the second equation?</p> <p>What does the 3 do in each equation?</p>   |   |   |           |            |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| <b>6</b> | <p>Equations like <math>F(x)</math> are called linear equations and have the form <math>Y =</math></p> <p>Equations like <math>G(x)</math> are called exponential equations and have the form <math>Y =</math></p> <p>Give the general form of each equation above, and state what each part of the equation represents.</p> |   |   |           |            |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

| 7   | <p>Some values for the functions <b>f</b> and <b>g</b> are shown in the table.<br/>One of the functions is linear. The other is exponential.<br/>Use the patterns to complete the missing entries.</p> <p>How does the Y-value change for each?</p> <p>Which function increases faster?</p> <p>Can you find an equation for each column?</p>   | <table border="1"> <thead> <tr> <th>X</th> <th>f</th> <th>g</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>2</td> <td>2</td> </tr> <tr> <td>1</td> <td>12</td> <td>20</td> </tr> <tr> <td>2</td> <td>22</td> <td>200</td> </tr> <tr> <td>3</td> <td>32</td> <td></td> </tr> <tr> <td>4</td> <td></td> <td></td> </tr> <tr> <td>5</td> <td></td> <td>200000</td> </tr> <tr> <td>6</td> <td>62</td> <td></td> </tr> <tr> <td>7</td> <td></td> <td></td> </tr> </tbody> </table> | X | f   | g  | 0 | 2 | 2 | 1 | 12 | 20 | 2 | 22 | 200 | 3 | 32 |  | 4 |  |  | 5 |  | 200000 | 6 | 62 |  | 7 |  |  |  |
|---|--|--|---|---|--|---|---|---|---|----|----|---|----|-----|---|----|--|---|--|--|---|--|--------|---|----|--|---|--|--|--|
| X   | f  | g  |   |   |  |   |   |   |   |    |    |   |    |     |   |    |  |   |  |  |   |  |        |   |    |  |   |  |  |  |
| 0   | 2  | 2  |   |   |  |   |   |   |   |    |    |   |    |     |   |    |  |   |  |  |   |  |        |   |    |  |   |  |  |  |
| 1   | 12   | 20   |   |   |  |   |   |   |   |    |    |   |    |     |   |    |  |   |  |  |   |  |        |   |    |  |   |  |  |  |
| 2   | 22   | 200  |   |   |  |   |   |   |   |    |    |   |    |     |   |    |  |   |  |  |   |  |        |   |    |  |   |  |  |  |
| 3   | 32   |  |   |   |  |   |   |   |   |    |    |   |    |     |   |    |  |   |  |  |   |  |        |   |    |  |   |  |  |  |
| 4   |  |  |   |   |  |   |   |   |   |    |    |   |    |     |   |    |  |   |  |  |   |  |        |   |    |  |   |  |  |  |
| 5   |  | 200000   |   |   |  |   |   |   |   |    |    |   |    |     |   |    |  |   |  |  |   |  |        |   |    |  |   |  |  |  |
| 6   | 62   |  |   |   |  |   |   |   |   |    |    |   |    |     |   |    |  |   |  |  |   |  |        |   |    |  |   |  |  |  |
| 7   |  |  |   |   |  |   |   |   |   |    |    |   |    |     |   |    |  |   |  |  |   |  |        |   |    |  |   |  |  |  |
| 8   | <p>Which scenario grows faster?</p> <table border="1" data-bbox="245 562 1458 636"> <tbody> <tr> <td data-bbox="245 562 849 636">A. You start with \$110 savings and add \$10 each week.</td> <td data-bbox="849 562 1458 636">B. You start with \$5 savings. Each week's total is double the amount you had the previous week.</td> </tr> </tbody> </table> <p>Write an equation for each and enter them in the input bar.</p> <p>In the table, enter the desired X-value. Then highlight both of the Y-value cells together and <b>FILL DOWN</b> to display both Y-values at the same time.</p> <p>How much money do you have after 1 week in each scenario? In 4 weeks?</p> <p>When does scenario B catch up with scenario A?</p> |  |   | A. You start with \$110 savings and add \$10 each week.                           | B. You start with \$5 savings. Each week's total is double the amount you had the previous week. |   |   |   |   |    |    |   |    |     |   |    |  |   |  |  |   |  |        |   |    |  |   |  |  |  |
| A. You start with \$110 savings and add \$10 each week.                           | B. You start with \$5 savings. Each week's total is double the amount you had the previous week.   |  |   |   |  |   |   |   |   |    |    |   |    |     |   |    |  |   |  |  |   |  |        |   |    |  |   |  |  |  |
| 9   | <table border="1" data-bbox="245 1073 1458 1146"> <tbody> <tr> <td data-bbox="245 1073 849 1146">A. You have a population of 100 guppies, which increases by 25 guppies each year.</td> <td data-bbox="849 1073 1458 1146">B. You have a population of 60 frogs, which increases by 25% each year.</td> </tr> </tbody> </table> <p>Write an equation for each.</p> <p>How many of each will there be in 3 years?</p> <p>When do the frogs outnumber the guppies?</p>   |  |   | A. You have a population of 100 guppies, which increases by 25 guppies each year. | B. You have a population of 60 frogs, which increases by 25% each year.                          |   |   |   |   |    |    |   |    |     |   |    |  |   |  |  |   |  |        |   |    |  |   |  |  |  |
| A. You have a population of 100 guppies, which increases by 25 guppies each year. | B. You have a population of 60 frogs, which increases by 25% each year.  |  |   |   |  |   |   |   |   |    |    |   |    |     |   |    |  |   |  |  |   |  |        |   |    |  |   |  |  |  |
| 10  | <table border="1" data-bbox="245 1465 1458 1539"> <tbody> <tr> <td data-bbox="245 1465 849 1539">Job A. Salary of \$25,000 with a 15% raise each year.</td> <td data-bbox="849 1465 1458 1539">Job B. Salary of \$25,000 with a \$5000 raise each year.</td> </tr> </tbody> </table> <p>Write an equation for each.</p> <p>When will Job A have a higher salary than Job B?</p> <p>Which would you rather have and why?</p>  |  |   | Job A. Salary of \$25,000 with a 15% raise each year.                             | Job B. Salary of \$25,000 with a \$5000 raise each year.   |   |   |   |   |    |    |   |    |     |   |    |  |   |  |  |   |  |        |   |    |  |   |  |  |  |
| Job A. Salary of \$25,000 with a 15% raise each year.                             | Job B. Salary of \$25,000 with a \$5000 raise each year.   |  |   |   |  |   |   |   |   |    |    |   |    |     |   |    |  |   |  |  |   |  |        |   |    |  |   |  |  |  |