Second degrees functions families

1) Let quadratic functions family: $f_m : \mathbb{R} \to \mathbb{R}$, $f_m(x) = mx^2 + 2(m+1)x + m - 1$, where m is real parameter.

a) Show that parabolas vertices belongs to line $y = x - 2$;

b) What part of this line contains vertices of parabolas whit branches up?

2) Let quadratic functions family: $f_m : \mathbb{R} \to \mathbb{R}$, $f_m(x) = x^2 - (2m-1)x + 4m + 3$, where m is real parameter. Show that parabolas f_m passing through a fixed point.

3) Let quadratic functions family: $f_m : \mathbb{R} \to \mathbb{R}$, $f_m(x) = x^2 - 2mx + 1$, where m is real parameter. Show that there are two parabolas tangents to x-axis. Then show that their vertices (ot this two parabolas) are simetrical to the point O.

4) Let quadratic functions family: $f_m : \mathbb{R} \to \mathbb{R}$, $f_m(x) = mx^2 - 2(m-2)x - (m+10)$, $m \in \mathbb{R}$.

a) Determine $m \in \mathbb{R}$, such that equation $f(x) = 0$ has real roots;

b) Determine $m \in \mathbb{R}$, such that $f(x) < 0$, $\forall x \in \mathbb{R}$.

c) Study function and draw graph for $m = 2$.

5) Let quadratic functions family: $f_m : \mathbb{R} \to \mathbb{R}$, $f_m(x) = (m-2)x^2 + (m+1)x + 4$, $m \in \mathbb{R}$

a) Find $m \in \mathbb{R}$, such that $f(x) = 0$ has no real roots;

b) Find $m \in \mathbb{R}$, such that $f(x) > 0$, $\forall x \in \mathbb{R}$;

c) Study function and draw graph for $m = 1$.

6) Let quadratic functions family: $f_m : \mathbb{R} \to \mathbb{R}$, $f_m(x) = mx^2 + (1-3m)x + 2m - 1$, $m \in \mathbb{R}$. Show that parabolas f_m passing through two fixed points.

7) Let quadratic functions family: $f_{m,n}:\mathbb{R}\to\mathbb{R}$, $f_{m,n}(x)=mx^2+2(m+n)x+m+2n$, $m\in\mathbb{R}^*, n\in\mathbb{R}$.

a) Show that, for fixed n, parabolas vertices belongs to a line.

b) Let *A, B* intersection points with x-axis, and F projection of vertex V to x-axis. Show that $\forall m\in\mathbb{R}^*,\ 2\cdot VF=|n|\cdot AB$.

c)) Show that all parabolas passing through a fixed point.

8) Let quadratic functions family: $f_m(x)=mx^2+2(m+1)x+m+2$, $m\in\mathbb{R}$, $m\in\mathbb{R}^*$.

a) Show that these parabolas vertices are on the line $y=x+1$.

b) Prove that all previously defined parabolas pass through a fixed point

9) Let quadratic functions family: $f_m:\mathbb{R}\to\mathbb{R}$, $f_m(x)=mx^2+2(m-1)x+m-1$, $m\in\mathbb{R}^*$.

a) a) Prove that these parabolas vertices are on the line $x+y=0$.

b) b) Prove that these parabolas pass through a fixed point.

10) Let quadratic functions family $f_m:\mathbb{R}\to\mathbb{R}$, $f_m(x)=(m+1)x^2-2(m+2)x+m+2$, $m\in\mathbb{R}$, $m\neq -1$.

a) Show that the parabolas passing through a fixed point .

b) Show that the vertices of these parables are on the line $x+y=0$.

c) Determine the portion of the line from b) that contains vertices of branches up parabolas.

d) Determine the parameter so that vertices of parabolas are:

1. above the x-axis.
2. on the x-axis.
3. to the right of y-axis.
4. on the line $y=1$.
5. under the line $y=-2$.