# MATH REFERENCE PAGES

| × | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| **2** | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 |
| **3** | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 | 42 | 45 | 48 |
| **4** | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 | 52 | 56 | 60 | 64 |
| **5** | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 |
| **6** | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 | 78 | 84 | 90 | 96 |
| **7** | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 | 91 | 98 | 105 | 112 |
| **8** | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 | 104 | 112 | 120 | 128 |
| **9** | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 | 117 | 126 | 135 | 144 |
| **10** | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 |
| **11** | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 | 143 | 154 | 165 | 176 |
| **12** | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 | 156 | 168 | 180 | 192 |
| **13** | 13 | 26 | 39 | 52 | 65 | 78 | 91 | 104 | 117 | 130 | 143 | 156 | 169 | 182 | 195 | 208 |
| **14** | 14 | 28 | 42 | 56 | 70 | 84 | 98 | 112 | 126 | 140 | 154 | 168 | 182 | 196 | 210 | 224 |
| **15** | 15 | 30 | 45 | 60 | 75 | 90 | 105 | 120 | 135 | 150 | 165 | 180 | 195 | 210 | 225 | 240 |
| **16** | 16 | 32 | 48 | 64 | 80 | 96 | 112 | 128 | 144 | 160 | 176 | 192 | 208 | 224 | 240 | 256 |
| **By** | | | **5th Grade** | | | | **6th Grade** | | | | **7th Grade** | | | | **8th Grade** | |

©ABC GED® TEST TIP Learning these tables well is **highly recommended** for GED® success! You will need to know common square roots to solve geometry problems about area and right triangles Each row of the multiplication table is the set of multiples for the row number. Two great tools to learn tables: (1) http://mathsbot.com/tools/timesTables (2) https://www.geogebra.org/m/x4qx9bz5 Knowing the factors of above values is a valuable asset for all math operations.

https://mathsbot.com/printables/timesTables **Practice Times Tables** ★★★

The **NCTM** recommends knowing to 12s by the 6th grade level. 9th graders in Algebra I will do better knowing the 16s (helps in GED studies).

***Knowing the prime factors of all the above products will be very valuable and important to your studies in Mathematics!***

The row or column values are the **Multiples** of a row's value.
A **Factor list** of numbers 16 {1,2,4,8,16} finds **GCF**.
**Factor pairs** of the numbers 16 {(1,16), (2,8), (4,4)}

## Prime Numbers ★★★

**Prime numbers** are important numbers which students should be familiar with! However, many tend to forget to use them. Very few student use them as often as they should. Why are they important?

1) Primes are unique, they are the product of a 1 and the prime number, and no other factors are available.
2) The prime factorization of a number is unique to every number.
3) Finding common denominators using primes can save time versus the LCM method.
4) Primes help find students determine if two or more numbers are relatively prime.
5) In algebra, understanding prime expressions allows one to solve complex expression fractions easier.

The Sieve of Eratosthenes helps to find the primes.
https://mathsbot.com/activities/sieveOfEratosthenes
The number "1" has only one factor and it is not a prime,
The zero and negative numbers are not prime.
The only even number that is prime is the common factor of all even numbers, i.e., "2"!
The GED/HSE exam writers expect you to know the first eight prime numbers, these numbers are: 2, 3, 5, 7, 11, 13, 17, 19, ...

**Sieve of Eratosthenes** a video explanation https://youtu.be/C_CHc66-Crk?si=avsiu178SI6vhayn
**Game to practice Primes:** https://mathsbot.com/puzzles/findThePrimes

**The primes in 1st 100 numbers**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | | 5 | | 7 | | | |
| 11 | | 13 | | | | 17 | | 19 | |
| | | 23 | | | | | | 29 | |
| 31 | | | | | | 37 | | | |
| 41 | | 43 | | | | 47 | | | |
| | | 53 | | | | | | 59 | |
| 61 | | | | | | 67 | | | |
| 71 | | 73 | | | | | | 79 | |
| | | 83 | | | | | | 89 | |
| | | | | | | 97 | | | |

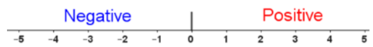
 

# MATH REFERENCE PAGES

## Signed Number Arithmetic (a.k.a., Integers, Rational, Real Numbers, Directed Numbers)

| | |
|---|---|
| + | Addition<br>Positive |
| – | Subtraction<br>Negative<br>Opposite |

***Addition***

Smaller Values  Larger values


Signs alike—*add the numbers, and keep the sign*

$(+) + (+) = (+)$ (**positive**) + (**positive**) = (**positive**)

$(-) + (-) = (-)$ (**negative**) + (**negative**) = (**negative**)

Signs different—*find the difference (subtract) between numbers and use sign of the apparent* ***larger value****, it is the distance between the values on the number line.

$\left.\begin{matrix}(+) + (-)\\ (-) + (+)\end{matrix}\right\}$ resulting sign depends on number having **greatest absolute value***. $\begin{cases}(\textit{positive}) + (\textit{negative})\\ (\textit{negative}) + (\textit{positive})\end{cases}$

***Subtraction***

Change all subtraction problems to an equivalent addition problem, follow addition rules.

Keep first number, change subtract to add, then add the opposite of the second number by the addition rules above.

a – b = a + (-b) This is read as: "***a minus b equals a plus the opposite value of b***".

To subtract signed numbers, **change sign of number being subtracted**, then **add to first number**.

**To add three or more signed numbers, add positive number, add negative numbers, then add two totals.**

***Multiplication*** ***and Division***

If the signs of the factors or the dividend and divisor are the same, the result is positive.

$(+)(+) = (+)$ (*positive*)(*positive*) = (*positive*)

$(-)(-) = (+)$ (*negative*)(**negative**) = (*positive*)

If the signs of the factors or the dividend and divisor are different, the result is negative.

$(+)(-) = (-)$ (**positive**)(**negative**) = (**negative**)

$(-)(+) = (-)$ (**negative**)(**positive**) = (**negative**)

Example of the patterns showing this rules are correct: https://www.youtube.com/watch?v=BEop4xwaFjU
Great practice for signed arithmetic: https://mathsbot.com/generators/directedMCQs
https://mathsbot.com/activities/wordedExpressions

## Basic Mathematical Knowledge Required

**Basic mathematical knowledge** needed to pass the HSE exams:

- Whole number arithmetic facts to the 16s, all factors of each product, square to 25, and cubes to 10
- Fraction arithmetic (05 Fraction Operations)
- Binary operations—two value operations with an arithmetic operator: plus (+), minus (–), times (×), divide (÷)
  - Value1 **Operator** Value2
- **Signed** (Directed) **Number** arithmetic—numbers with conditional location symbols: positive (+) or negative (–)
  - opposite is a conditional modifier—opposite (–) changes a sign to its opposite condition,
  - negative become positive and positive becomes negative.
- **Absolute Value***—the value of a signed number without any conditional symbols: $|n| = \begin{cases} n, & \text{if } n \geq 0 \\ -n, & \text{if } n < 0 \end{cases}$
  - If a question refers to a **distance** or **implies a distance**, the question is asking for the **absolute value** in the solution.
  - $|-5| = 5$ or $|5| = 5$ {"absolute value of negative five equals five" or "absolute value of five equals five"}
  - $|-7|$ ___ $|-9|$, $|-9|$ ___ $|7|$

Many get the following mathematical statements incorrect by not using the correct vocabulary/syntax:

| Problem | Verbal Math Statement (use shows you understand) | Using signed numbers |
|---|---|---|
| 5 + 6 = 11 | five plus six is eleven | 5 + 6 = 11 |
| 5 + –6 = -1 | five plus the opposite of six is **negative** one | 5 – 6 = –1 |
| 5 – 6 | five **minus** six is five plus **negative** six is **negative** one | 5 – 6 = **5 + (-6) = -1** |
| 5 – –(6) | five **minus** the **opposite** of six | 5 – –(6) = 5 + 6 = 11 |
| 5 – (–6) | five **minus** **negative** six | 5 – (–6) = 5 + 6 = 11 |
| 5 – –(–6) | five **minus** the **opposite** of **negative** six | 5 – –**(–6)** = 5 – **6** = –1 |

**Operator** Examples:
Plus: 3 + 7 Minus: 7 – 3
Multiply: 3 × 7 Divide: 21 ÷ 7

**Condition** Examples:
Positive ⁺8 Negative ⁻7

**Condition Modifier** Examples:
Opposite of 8: –(⁺8) = ⁻8
Opposite of ⁻9: –(⁻9) = ⁺9

**Learning the proper vocabulary for mathematics in the use of the plus (+) or minus (–) symbols will assist proper understanding of mathematical symbols and understanding.**

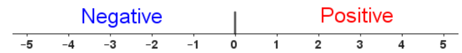
# MATH REFERENCE PAGES

How a sign is used in a mathematical phrase sets the meaning of that sign and **how you name the sign**.

**Plus**, what does it mean?

The *plus* (+) sign can mean different things, depending on the context.

- It means to add the two values which are separated by it, a binary operation. 5 + 8 or (five plus eight)
- It is a condition of being a **positive** number which is on the right-hand side of zero on a number line. Written positive signs are optional, i.e., $^{+}22$ and 22 are equivalent; **positive** is a condition.




**Minus**, what does it mean?

The *minus* (–) sign can mean three different things, depending on the context.

- It means to **subtract** the two values separated by it. Between two expressions, it means subtract the second expression from the first one. For example, x **–** 3 means x minus three. **It is a binary operation, not a condition.**
- It is a condition of being a **negative** number which is on the left-hand side of zero on a number line. Example: $^{-}2$ means negative 2. Negative numbers **require** a negative sign; **negativity** is a condition.
- Or it is a condition modifier, asking for the **opposite** of the value current condition. The opposite of a number is what you add to it to get zero. Example: –2 can mean the opposite of 2, which is negative 2, since 2 + -2 = 0. Likewise, -x means the opposite of x, and x + -x = 0.
  - This third condition of opposite allows one to change any subtraction problem into an addition problem. Meaning that we can apply certain freedoms to arithmetic the subtraction does allow. Everyday usage: **a - b = a + (-b)** is a reminder for students of the rule.

Adapted from: *Algebra: Themes, Tools, Concepts* © 1994 Anita Wah and Henri Picciotto

| Operator (binary) | Description | Operator | Description of Unary Operator |
|---|---|---|---|
| + | Addition Operator | + | Unary positive operator; indicates positive value (opt.) |
| – | Subtraction Operator | – | Unary negative operator; negates an expression (opposite) |
| × | Multiplication Operator | ++ | Increment operator; increments a value by 1 |
| ÷ | Division Operator | – – | Decrement operator; decrements a value by 1 |

Shaded portions not tested. *https://docs.oracle.com/javase/tutorial/java/nutsandbolts/op1.html*

## Absolute Value, Coefficients, Exponents, and Expressions

### Coefficient Concepts

The coefficient is the number of times a variable (value) or groups of variables are added in a list of the same values.

3x = x + x + x

5x = x + x + x + x + x

3x + 5x = x + x + x + x + x + x + x + x

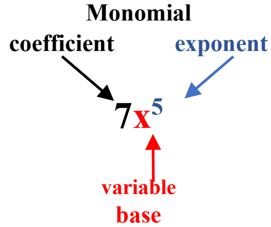
3x + 5x = 8x

5x - 3x = x + x + x + x + x

5x - 3x = 2x

3 🍎 +5 🍎 =8 🍎 and 8 🍐 – 5 🍐 = 3 🍐

### Monomial




Monomials can be a single number, a single variable, a number followed by 1 or more variables:

- **The coefficient is the number multiplied by 1 or more variable(s) and/or its exponent. The coefficients of like monomials follow the rules of arithmetic.**
- **An exponent is the number of times the base is repeatedly multiplied.**

All single variables have a coefficient of 1 and an exponent of 1 which are not normally displayed.

$$x = 1 \bullet x^1$$

Unwritten coefficients or exponents have a value of **one**, **1**.

All monomials are **terms**, the building blocks of **expressions**.

### Exponent Concepts

An exponent is the number of times an element of the exponent is multiplied within the list of the same elements.

$x^3 = x \bullet x \bullet x$

$x^5 = x \bullet x \bullet x \bullet x \bullet x$

$x^3 \bullet x^5 = x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x$

$x^3 \bullet x^5 = x^{3+5} = x^8$

$x^5 \div x^3 = \frac{x \bullet x \bullet x \bullet x \bullet x}{x \bullet x \bullet x} = x^2$

$x^5 \div x^3 = x^{5-3} = x^2$

### Absolute Value

Definition:

$$|b| = \begin{cases} b, \; b \geq 0 & non-negative \\ -b, \; b < 0 & negative \end{cases}$$

Absolute Value is a **distance**, if the problem mentions distance specifically or by inference a distance. Basically, it removes the condition modifier signs from all numbers. The values are neither positive nor negative, they are the distance between values.

$$|^{+}7| = 7 \text{ and } |^{-}7| = 7$$

If a number has a sign (+ or –), it is a directed number. Directed numbers are a distance from zero on the number line. **Absolute values have no direction**.

### Expressions

Examples of sum(s) and/or difference(s):

**Monomial Expressions** have no + or – signs.

**Binomial Expressions**:

x + y, sum of monomials

x – y, difference of monomials

**Trinomial Expressions**:

x + y + 3, all sums

x + y – 3, a sum and a difference

x – y + 3, a difference and a sum

x – y – 3, all differences

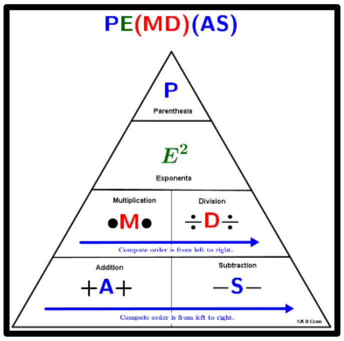
**An expression has n – 1 operators for its n-terms.**

### Equations and Inequalities

An **equation** has an expression = to another expression. An **inequality** has an inequality sign… <, >, ≤, ≥, or ≠… replacing the =.

# MATH REFERENCE PAGES




| P | E | (M D) | (A S) |
|---|---|---|---|
| Parenthesis | Exponents | × or ÷ | + or − |
| { } [ ] ( ) \| \| Implicit uses | $m^n$ $\sqrt[n]{m}$ | × or ÷ → | + or − → |

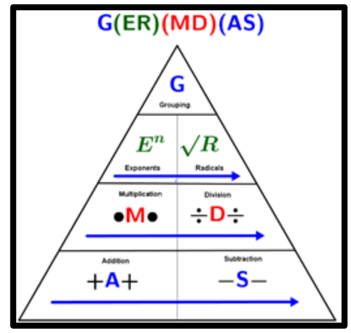
A B Cron (c)2023

Operations in parenthesis are done in **left-to-right** order as read.

| Grouping | Exponents or Roots | × or ÷ | + or − |
|---|---|---|---|
| G | (E R) | (M D) | (A S) |

**Implicit uses**: expressions in dividends, radicands, absolute value, numerators, denominators, exponents, etc.

**The major groups are Grouping, Exp/Roots, Mult/Div, Add/Sub (GEMA).**
**The process is do all operations within groups following EMA.**
**When all groups are gone perform EMA on remainder.**
The sign of a signed number has the lowest precedence.




# Operations, Powers, and Roots (Radicals)

**Addition**:
5 + 8 = 13
addend + addend = sum

**Subtraction**:
13 – 8 = 5
13 – 5 = 8
subtrahend – minuend = difference

**Multiplication**:
7 × 8 = 56
factor × factor = product

**Division**:
56 ÷ 8 = 7
56 ÷ 7 = 8
quotient ÷ factor = factor

**Power**:

$$7^2 = 49$$

$$base^{exponenet} = power$$

or

7 ^ 2 = 49
base ^ exponent = power

**Root**:

$$\sqrt[2]{49} = 7 \text{ or } 49^{\frac{1}{2}} = 7$$

$$\sqrt[degree]{radicand} = root$$

There are four (4) basic operations used since elementary school:

1) An addition operation (+) combines two values into one value.
   a. The values are called addends.
   b. The result is called the sum.
   c. Every sum can be reversed two (2) ways, via subtraction.
2) A subtraction operation (–) extracts the difference of two values as one value.
   a. The subtraction values, the subtrahend and minuend, result in the difference.
   b. The subtrahend was the sum of the addition problem, and the minuend and difference were the addends.
3) A multiplication operation (×) combines two values which are the equivalent to ‘a’ times ‘b’ set.
   a. The values multiplied are called factors.
   b. The multiplied value is called the product.
   c. Every product can be reversed two (2) ways, via division.
4) A division operation (÷) extracts the quotient of two values as one value.
   a. The division value, the dividend and divisor, result in the quotient.
   b. The dividend was the sum of the multiplication problem, and the divisor and quotient were the factors.
5) A power or exponential operation (m^n or $m^n$) takes a base and repeats it the value of the exponent’s value.
   a. A power value is the product of the base as many times as the exponent’s value (see above Vital Concepts II).
   b. An exponent determines the degree of the base’s value.
6) A root value ($\sqrt{n}$, $\sqrt[m]{n}$, $n^{\frac{1}{m}}$ ) extracts the root from the degree of the radicand.
   - The root is a single value when multiplied by itself the value of the degree (exponent) is the radicand.

When do students normally get introduced to:
Geometry and Number Lines: Kindergarten to present
Addition and Subtraction: K – 2nd grades
1st Informal Intro to Algebra 3 + 🍓 = 8, what is 🍓 ?
Parentheses: 3rd grade to change order of addition/subtraction
Multiplication and Division: 3rd and 4th grades
2nd Informal Intro to Algebra 3 × 🍓 = 18, what is 🍓 ?
Exponents and Roots: 5th and 6th grades
Fractions are ongoing from K, operations within 3rd and 4th
Percentages in 3rd. Decimals: 4th or 5th grade
Signed numbers: 5th or 6th grade
Variables: 5th grade
Algebra: 6th grade, more on letters as variables.

| **Integer Exponents, $m^n$**: | **Fractional Exponents**: the numerator is the power index, and the denominator is the radical index. |
|---|---|
| n>0: you multiply the number times itself n times; $5^1 = 5$; $5^2 = 5 \times 5$; … $5^4 = 5 \times 5 \times 5 \times 5$ | $value^{\frac{power\ index}{radical\ index}}$ |
| n=0: the value is 1 if the base≠0; $7^0 = 1$; $225^0 = 1$; $0^0$ undefined | $8^{\frac{2}{3}} = \sqrt[3]{8^2}$ |
| n<0: this means the reciprocal of the value; $3^{-1} = \frac{1}{3}$, $256^{-1} = \frac{1}{256}$, $4^{-2} = \frac{1}{4} \times \frac{1}{4}$, $25^{-3} = \frac{1}{25} \times \frac{1}{25} \times \frac{1}{25}$ | $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$<br>$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$ |

# MATH REFERENCE PAGES

## The Rules of Divisibility

**Divisibility Rules:** (means the number will divide by the stated value)

adjective: **divisible**

capable of being divided by "the marine environment is divisible into a number of areas"

- ***Mathematics***: (of a number) capable of being divided by another number without a remainder. "36 **is divisible by** {1, 2, 3, 4, 6, 9, 12, 18, 36}" [This is the list of **factors of 36**, only these factors divide 36.][1]

**Divisibility Rules** are used to help one with division of numbers, fractions, and factoring of algebraic expressions. Many learned some of the rules are learned in the elementary school when your teacher had you counting by twos, threes, fives, and tens. **All numbers** (variables) **have a common factor of one, 1.**

**Divisible by 2** ★★

If a number is **even**, it is divisible by **2**. But what is even, the number ends in one of the following values: 0, 2, 4, 6, 8. These are the values when counting by twos; all even numbers are divisible by two.

**Divisible by 5**★★

If a number ends in 5 or 0, it is divisible by 5. These are the values when counting by five.

**Divisible by 10**

If a number ends in 0, it is divisible by 10. These are the values when counting by ten.

The above values are the quickest to learn/recall as you counted by 2s, 5s, and 10s during first grade. The most commonly used divisibility rules are **2, 3, and 5** (they are expected HSE knowledge). While these three numbers are the first three **prime numbers** (see following pages). The divisibility test for 3 is least recalled.

**Divisible by 3** ★★

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Digits sum: | 3 | 6 | 9 | 3 | 6 … | | 9 | 9 | 12 | 18 … |
| Digits: | … 12, | 15, | 18, | 21, | 24, | … | 126, | 315, | 633, | 936, … |

If a number is divisible by 3, there is little trick to verify divisibility. This trick is to find the sum of all the digits of a number. If the sum is a multiple of 3 (or divides by 3), the number as written divides by 3.

For example:

1) 125 has 3 digits, add 1+2+5 = 8; is 8 divisible by 3? No. 125 does not divide by 3!
2) 2514 has 4 digits, add 2+5+1+4 = 12; is 12 divisible by 3? Yes! 2514 divides by 3, 838 times!
3) 45,372,123 divides by. Add its digits: ____ Does that sum divide by 3?

https://www.youtube.com/watch?v=_3reREWx5K8&pp=ygUYaG93aWUgaHVhIGRpdmlzaWJsZSBieSAz

Recap: (The underlined values are HSE **expected** knowledge.)

**Divisible by 2:** last digit is even; 0, 2, 4, 6, 8
**Divisible by 3:** sum of digits divides by 3
Divisible by **4:** last two digits divide by 4; 00, 04, 08, 12, 16, 20, 24, …, 92, 96
**Divisible by 5:** ends in 5 or 0
Divisible by **6:** an even number divisible by 3; or divisible by 2 and 3
Divisible by **7:** Double the last digit and subtract it from a number made by the other digits. The result must be divisible by 7. (We can apply this rule to that answer again.)
Divisible by **8:** last 3 digits divide by 8
Divisible by **9:** sum of digits divisible by 9
Divisible by **10:** ends in 0

| | | |
|---|---|---|
| 2 THANK | 3 YOU | 4 ALL |
| 5 FOR | 6 HAVING | 7 NOT YOU |
| 8 EASY | 9 DIVISIBILITY | 10 RULES |

Online Resource:
**https://www.mathsisfun.com/divisibility-rules.html**

[1]See 01 Translating English Words to Algebra

**Square root of negative values?**
**FYI: Not a tested item on HSE, next course level.**

- -1 is a factor of every negative number (as is 1).
- The square root of -1, is called $\sqrt{-1} = i$ an imaginary number. Complex numbers are real numbers add to imaginary numbers. $4 + 3i$ & $4 - 3i$; when the Discriminant < 0.
- Used in understanding electricity and electronic designs.

# MATH REFERENCE PAGES

## Fractions vs Decimals

**Decimals** are base-ten positional numeral system. While decimal systems have been in use for over 2000 years, modern methods were invented less than 500 years ago. They did not become in common usage until after 1790 when the French mandated the metric system in France. **Rational decimals** can be represented by decimal fractions of the form $\frac{a}{10^n}$, where "*a*" is an integer, and "*n*" is a non-negative integer. Rational decimals are either **terminating** or **repeating**. All rational decimals can be represented in a reduced fraction form. If a decimal number cannot be represented in reduced fraction form, it is called an **irrational decimal**, and the decimal value never terminates or repeats. Together, these are **real numbers**.

| Terminating Decimals | | Repeating Decimals | | Irrational Decimals | |
|---|---|---|---|---|---|
| Decimal | Fraction | Decimal | Fraction | Number | Decimal |
| 0.5 | $\frac{1}{2}$ | 0.333333333… | $\frac{1}{3}$ | π | 3.14159265359… |
| 0.375 | $\frac{3}{8}$ | 0.142857142… | $\frac{1}{7}$ | $\sqrt{3}$ | 1.73205080757… |
| 3.3125 | $3\frac{5}{16}$ | 5.272727272… | $5\frac{3}{11}$ | $\sqrt[3]{25}$ | 2.92401773821… |

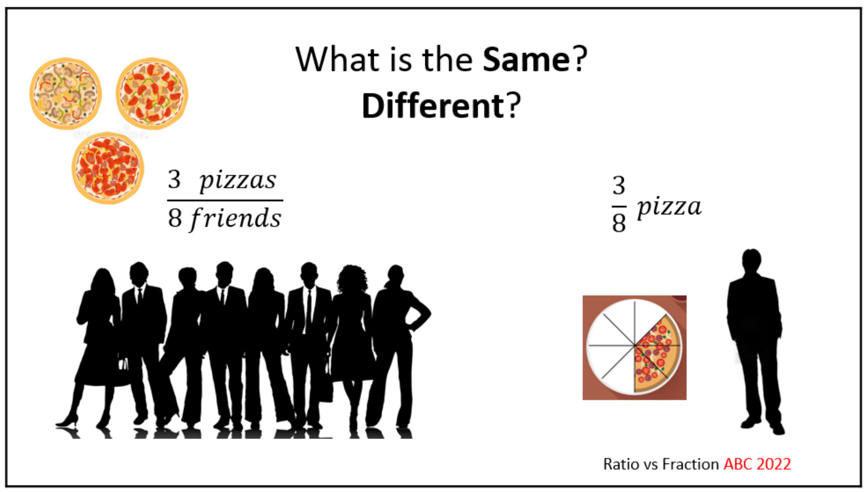
## Fractions, Ratios, and Rates

"Fractions are your friend!"

**Fractions** compare "*like items*" where you have the number of parts over the whole number of parts. (Fractions a.k.a., *rational numbers*, are formed by any integer over any non-zero integer.)

$\frac{part}{whole}$ – compares like items
(fraction)

A **ratio** is a comparison of 2 or more numbers. The numbers may be *like or unlike items*, the items in the numerator may not be the same type of items in denominator.




people : animals – comparing unlike items (ratio)

birds : grains of sand in the air or $\frac{birds}{grains\ of\ sand}$

**Fractions and ratios only use integers, only rates can have decimals values.**

Hence, "All **ratios** are fractions, but not all **fractions** are ratios."

All ratios must always be in fraction form unless they represent a **Rate**. Driving 135 miles with 6.5 gallons of gas is $\frac{135 \div 6.5}{6.5 \div 6.5} = 20.8\ mpg$ . The rate is miles per hour (mph) or miles per gallon (mpg). For rates, the denominator is always 1. **Rate compare something to one thing**…it is a special ratio, the units allow the user to not use a 1 in the denominator: mpg, mph. But somethings cannot be divided, if I have 8 people and 3 pizzas the fraction would $\frac{8}{3}$ or 8:3, but not 2.67 people per pizza.

miles : gallon — compares fuel consumption per mile (rate)

**"There are some fraction rules that ratios do NOT follow. Do not change a ratio that is an improper fraction to a mixed number. Also, if a ratio in fraction form has a denominator of 1, do not write it as a whole number. Leave it in fraction form." Kaplan GED, p 260.**

**Proportions** occur whenever a ratio is equivalent to another ratio. $\frac{in}{foot} = \frac{yards}{mile}$

Keep this Page for Review

# MATH REFERENCE PAGES

## Adding and Subtracting Fractions★★★★★

4 Methods to find a **Common Denominator**

Steps for finding common denominators needed for **addition** or **subtraction**:

1. Are the denominators the same (like denominators)?
   A. Yes, go on add/subtract numerators and **reduce** to lowest terms when possible. $\frac{8}{35} - \frac{3}{35} = \frac{5}{35} = \frac{1}{7}$
   B. No, go to step 2.
2. Does one of the denominators divide the other one?
   A. Yes, go on multiply the smaller values fraction parts by the quotient. $\frac{7}{12} - \frac{1}{3}; 12 \div 3 = 4$
   - 12 divides by 3 four times, the result **4** is multiplied by both parts of the fraction with the smaller denominator: $\frac{7}{12} - \frac{1 \times 4}{3 \times 4} = \frac{7}{12} - \frac{4}{12} = \frac{3}{12} = \frac{1}{4}$
   - Subtract or Add, then **reduce if possible**.

   B. No, go to step 3.
3. Do the denominators have any common factors? LCM (LCD) of the denominators by one of two methods.
   A. Write the multiples of each denominator until you find the lowest common multiple.

   1 2 3 4 5 6 7 8 9 are the multipliers for each fraction.

   Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, … (24 is the 4th number)

   Multiples of 8: 8, 16, 24, 32, 40, 48, 56, … (24 is the 3rd number)

   While there are many common multiples, the lowest one is **24**. So, using the 24 as a denominator, multiply the 5 by 4 and multiply the 7 by 3 to make the fractions with common denominators.

   Add or Subtract numerators, then **reduce if possible**.

   $$\frac{5}{6} + \frac{7}{8} = \quad \frac{5 \cdot 4}{6 \cdot 4} + \frac{7 \cdot 3}{8 \cdot 3} = \quad \frac{20}{24} + \frac{21}{24} = \quad \frac{41}{24} = 1\frac{17}{24}$$

   B. Find prime factorization of each one denominator, determine any common factors (Greatest Common Factor, GCF), determine the lowest common multiple (denominator); **reduce** **if possible.** {Ladder Method or Factor Tree}

   $$\frac{5}{8} + \frac{7}{18}$$

   | | |
   |---|---|
   | 2 \| 8 | 2 \| 18 |
   | 2 \| 4 | 3 \| 9 |
   | 2 | 3 |

8: {1, 2, 4, 8} prime numbers

18: {1, 2, 3, 6, 9, 18}

The prime factorization of 8: 2 × 2 × 2

The prime factorization of 18: 2 × 3 × 3

Lowest Common Multiple is: 2 × 2 × 2 × 3 × 3 = 72, the highlighted part is 8, the underscored is 18, and the GCF is 2 as the shared factor which we do not use in new fraction forms.

LCM(LCD) method

Multiply the 5 & 6 by 2 x 2; or 5 × 4 & 6 × 4

Multiply the 7 & 18 by 3; or 7 × 3 & 18 × 3

Write the factors of the denominators

1 2 3 4 5 6 7 8 9 10 multiply by

8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80 …

18: 18, 36, 54, 72, 90, …

https://www.geogebra.org/m/j4UyPdKW#material/xSatv2V9

Select the multiplier in light blue above LCM.

$$\frac{5 \cdot 9}{8 \cdot 9} + \frac{7 \cdot 4}{18 \cdot 4} = \frac{45}{72} + \frac{28}{72} = \frac{73}{72} = 1\frac{1}{72}$$

   C. No, go to step 4.

4. If either of the denominators is a **prime** (or is relatively prime, meaning they share no common factors), multiply fraction parts (numerator & denominator) by its unshared factor.
   A. If any denominator is a **prime number**, multiply each denominator by the numerator and denominator of the other fraction.
   B. If denominators are not prime number and do not share any factors other than 1, the denominators are **relatively prime** to each other. Hence, multiply each denominator by the numerator and denominator of the other fraction.
   - The *factor pairs* of 12 are {1,12}, {2,6}, {3,4} or *factor set* = {1,2,3,4,6,12}. The **prime factorization is 2•2•3.**
   - The factor pairs of 25 are {1,25},{5,5} or factor set = {1,5,25}. The **prime factorization is 5•5.**

   Since the factor sets share no factors other than 1, they are **relatively prime**, hence multiply each denominator by both parts of the other fraction for all of the fractions used. Add or Subtract numerators, then **reduce if possible**.

**Prime Factor**

$$\frac{3}{5} - \frac{4}{7} = \quad \frac{3 \times 7}{5 \times 7} - \frac{4 \times 5}{7 \times 5} = \quad \frac{21}{35} - \frac{20}{35} = \frac{1}{35}$$

**Relatively Prime**

$$\frac{5}{12} + \frac{12}{25} = \quad \frac{5 \times 25}{12 \times 25} + \frac{12 \times 12}{25 \times 12} = \quad \frac{125}{300} + \frac{144}{300} = \frac{269}{300}$$

# MATH REFERENCE PAGES

## Simplifying an Algebraic Expression

All algebraic expressions are made up of monomials (p. 3). <u>3 + 5</u>, 3 + 5x, 3y + 5, and <u>3x + 6x</u> are expressions, only the 1st and last can be simplified as 8 and 9x. The rest do not have like terms and cannot be simplified.

**Simplifying Expressions** (a <u>prerequisite</u> for solving equations and inequalities)

Which of the following operation can be performed: 3🍏 + 4🍓, 3🍏 +4🍏, or 3🍓 + 2🍓 - 5🍏?

Why can you add some, but not all of them? ______________________________

Only items that are alike can be added or subtracted. It does not matter if it is different fruit or variable (letters in the alphabet.) So 3x + 4y cannot be added since x and y are different, but 3x + 4x can be added to be 7x. They are like terms (fruits).

The basic rules for adding and subtracting like terms are the variable parts of a term must be **identical**. If the terms are <u>different</u>, we <u>canno</u>t add or subtract them. The rules for multiplication and division are very different. They will discussed once you have learned about linear expressions and equations.

The <u>goal for simplifying any expression</u> is to combine all **like terms**, a simplified expression is one where all the terms in the expression have all coefficients combined for each unique variable term.

**3x + 5 – 6x – 8  + 12x + 31** since there is only addition and subtraction, we can group like terms keeping signs.

3x – 6x + 12x + 5 – 8 + 31,  3x – 6x + 12x is 9x, 5 – 8 + 31 is 28, resulting in 9x + 28.

**3x + 5y + 7 – 6y – 3 + 5x**, 3x + 5x + 5y – 6y + 7 – 3, result is 8x – y + 4.

The above are simple examples of linear expressions. On careful examination of the problems, you will find that the reorganization of the terms kept the sign of the original term. At first this may seem like a violation of the Order of Operations, but it is a feature of working with positive and negative numbers: 5 – 4 = 5 + (-4).

While the above were simple examples of **linear expressions**, in essence quadratic, cubic, and other expression follow the exact same rules.

### <u>Basic Linear Equations (Inequalities) in One Variable</u> ★★★★★

Whenever an equal sign is placed between two linear expressions, the result is a **Linear Equation.**

Rules for simplifying linear equations in one variable:

1. Simplify each side's linear expression. If you do this with single variable equation (inequalities), you set yourself up for 2 to 4 initial addition or subtract choice(s).  {a x + b = c x + d}
2. If the any the values of a, b, c, or d equals a zero, this reduces the original choices by one. However, if both sides had variables it is works out to be the 2 constant choices.
3. If you have a variable value on both side, either Add or Subtract the variable term so that the resulting variable part on either side of the equation has a positive coefficient.
4. Now, Add or Subtract the constant term with the variable part to both sides of the equation. The constant part is now alone of the other side of the equation.
5. If the variable term has a coefficient different from 1, multiply or divide both sides by the coefficient.
6. The result is the value of the variable.

$$5x + 15 - 2x = 14 - 8x - 7$$
$$3x + 15 = 7 - 8x$$
$$3x + 15 + 8x = 7 - 8x + 8x$$
$$11x + 15 = 7$$
$$11x + 15 - 15 = 7 - 15$$
$$11x = -8$$
$$\frac{11}{11}x = \frac{-8}{11}$$
$$x = -\frac{8}{11}$$

**Not all lines above are written for every problem. 4th and 6th optional**

### Basic Linear Equations in Two Variables ★★★★★

Whenever a linear equation has two variables, the equation can be simplified into several basic forms depending on the use intended for them. The linear equations at the right are the most common forms of linear equations. <u>The slope-intercept form is commonly used on the HSE exams for multiple and various questions.</u> There are two additional formulas, one is the slope, and the other is the Point-Slope form which is designed to assist in getting the value of m when you have <u>two points</u> and b when you only or a <u>slope and single point</u>.

Using the process of solving simple linear equations in one variable, modify it to solve for the one of the two variables in the equation, usually the 'y'.

# MATH REFERENCE PAGES

## Multiplication Practice Page

Print out several copies. Fill out each row as fast as you can by counting by the **row or column numbers**. Repeat as often as you need to do until you learn them all. Perfection occurs when you can look at a number in the grid and state the factors (row/column headers). Practice times tables often. https://mathsbot.com/printables/timesTables

| × | 1 | **2** | **3** | 4 | **5** | 6 | **7** | 8 | 9 | 10 | **11** | 12 | **13** | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | | | |
| **2** | | | | | | | | | | | | | | | | |
| **3** | | | | | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | | | | | |
| **5** | | | | | | | | | | | | | | | | |
| 6 | | | | | | | | | | | | | | | | |
| **7** | | | | | | | | | | | | | | | | |
| 8 | | | | | | | | | | | | | | | | |
| 9 | | | | | | | | | | | | | | | | |
| 10 | | | | | | | | | | | | | | | | |
| **11** | | | | | | | | | | | | | | | | |
| 12 | | | | | | | | | | | | | | | | |
| **13** | | | | | | | | | | | | | | | | |
| 14 | | | | | | | | | | | | | | | | |
| 15 | | | | | | | | | | | | | | | | |
| 16 | | | | | | | | | | | | | | | | |

There are many advantages in learning your multiplication tables to the 16s.

(1) Each row & column is the list of **multiples** of each number, which will help with finding **Lowest Common Denominator** for fraction addition & subtraction problems.

(2) Many test writers use these factors repeatedly in writing common problems.

(3) The math & algebra on HSE (GED®) will be easier knowing these facts (do not memorize, learn them by using them.)

(4) The chart title numbers in **bold blue** are the **prime numbers**. Prime numbers are the building blocks of composite numbers.

(5) While many people think math is a jumble of subtopics without rhyme or reason, in reality, math is an integrated system of concepts which make the world about your work.

The more you embrace math skills, the better life in the world becomes for you and your family.

5 tips for success in math class by Howie Hua.


Learn how to count the number of factors a value has:

How many factors does 72000 have?
https://www.youtube.com/watch?v=Zp6jzQFtawk

Where does the percent sign (%) come from:
https://en.wikipedia.org/wiki/Percentage

Use for fractions operations, factoring expressions, factoring equations, and inequalities:

Factor pairs: 12: {(1, 12), (2, 6), (3, 4)} 16: {(1, 16), (2, 8), (4, 4)}

Factor sets: 12: {1, 2, 3, 4, 6, 12} 16: {1, 2, 4, 8, 16}

Prime factors: 12: $2 \times 2 \times 3$ 16: $2 \times 2 \times 2 \times 2$

Using a comparison of prime factor sets gives the:

LCM(12,16): $2 \times 2 \times 2 \times 2 \times 3 = 48$ GCF(12,16) = $2 \times 2 = 4$

# MATH REFERENCE PAGES

Use grid paper, write random numbers in no particular order from 1- up to 16 across top and vertically.

To practice the values which are giving you trouble arrange the values in a random order on grid paper. See on the right:

Example:
Horizontally: 2, 5, 7, 3, 10, 12, 8, 9, 0, 13, 6, 15
Vertically: 4, 6, 9, 3, 11, 16, 1, 8, 5, 12, 2, 14

https://mathsbot.com/printables/timesTables

**Select range of numbers and Jumble them** ★★

| × | | | | | | | | | | | | |
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Remember these facts:

A. **Prime numbers** have exactly two factors, one factor is the number 1 and the other factor is that number. 5 = 5 x 1 or 1 x 5 = 5

B. **Composite numbers** have more than two factors.

C. All equations and inequalities sentences have the syntax at the right: →

| *Expression* | *Verb* | *Expression* |
|---|---|---|
| *3x + 1* | = | *5x – 7* |
| *3x + 1* | ≤ | *5x – 7* |

D. Basic arithmetic rules (laws) **These all apply to every mathematical arena.**

a. Commutative Properties: a + b = b + a or ab = ba

b. Associative Properties: a + (b + c) = (a + b) + c or a(bc) = (ab)c

c. Identity Properties: a + 0 = a or a × 1 = a

d. Inverse Properties: a + (-a) = 0 or $a \times \frac{1}{a} = 1, if\ a \neq 0$

**Signed Number Arithmetic: a – b = a + (-b); uses the additive inverse property**

**Simplifying fraction division: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$; uses the reciprocal property: $\frac{a}{b} \times \frac{b}{a} = 1$**

**e.** Distributive Property: a(b ± c) = ab ± ac; reversing this is called factoring where we do this:
ab ± ac = a(b ± c). **(± means you work with either + or –.)**

f. Multiplicative Property of Zero: a × 0 = 0

E. **Remember:**

a. **Addition and Subtraction are inverse operations!** **Each addition has two subtractions.**

b. **Multiplication and Division are inverse operations!** **Each multiplication has two divisions.**

c. **Exponents and Roots are inverse operations!** **Each exponential has a single root.**

d. All functions are relations, but **NOT** all relations are functions.

e. Knowing the number of factors of any value is a useful tool. The **prime factorization** of 36 = $2^2 \times 3^2$, so it has 9 factors, i.e., {1,2,3,4,6,9,12,18,36}. The product of each exponent + 1 from the prime factorization solves this. The exponents of 36's prime factorization are both 2, so 2+1=3, therefore, 3×3=9 factors. $360 = 2^3 \times 3^3 \times 5$, so there are 4×4×2=32 factors of 360.

f. **The "+" and "–" symbols have multiple uses and names:**
When the symbols are between two values (numbers), the values are added, **plus**, or subtracted, **minus**. However, if they precede a single value, they are either a **positive** or a **negative** location on a number line. The "–" symbol, **opposite**, can be used to change a-positive-to-a-negative or a-negative-to-a-positive value, i.e., it returns the opposite of the value.

**Algebra uses all these Arithmetic rules.**

 

# MATH REFERENCE PAGES

**Origins of Per Cent Sign**

The word "percent" comes from the Latin phrase per centum, which means "by the hundred" or "hundred". The word was first recorded in English in the 16th century. The symbol for percent, "%", comes from the Italian term per cento, which means "for a hundred". The "per" was often abbreviated as "p" and eventually disappeared, and the "cento" was contracted to two circles separated by a horizontal line.

The word "percent" can be used as a noun or an adjective:

- **Noun**: One one-hundredth part, or 1/100. For example, "a 16 percent decline".
- **Adjective**: Figured or expressed on the basis of a rate or proportion per hundred. For example, "to get three percent interest".

*From <[https://www.google.com/search?client=firefox-b-1-d&q=Origins+of+Per+Cent](https://www.google.com/search?client=firefox-b-1-d&q=Origins+of+Per+Cent)>*

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How and where might you use ratio tables? (Extension of Page 6 on Ratios/Fractions)

$15 \times 5 = ? \times 25$ Simplify $15 : 25$

Simplify $\frac{15}{25}$ Solve $3 : 5 = 15 : x$

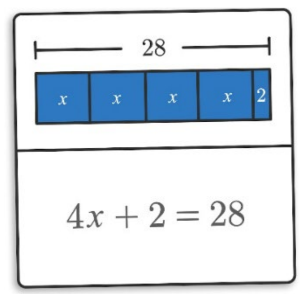
| | |
|---|---|
| Ana and Bill share sweets in the ration 3:5. Bill takes 25 sweets. How many does Ana have? | If 5% of $25, what is 2%? |

| | |
|---|---|
| Two identical pies cost $15. How much would five pies cost? | Ted runs 15 miles in three hours, how long might it take to run 25 miles? |

| | |
|---|---|
| 3 | 15 |
| 5 | 25 |

---

Tape diagram of an algebraic equation.



# MATH REFERENCE PAGES

**Learn your Multiplication Tables 4 Success in Life**

| × | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 | 42 | 45 | 48 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 | 52 | 56 | 60 | 64 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 | 78 | 84 | 90 | 96 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 | 91 | 98 | 105 | 112 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 | 104 | 112 | 120 | 128 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 | 117 | 126 | 135 | 144 |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 |
| 11 | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 | 143 | 154 | 165 | 176 |
| 12 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 | 156 | 168 | 180 | 192 |
| 13 | 13 | 26 | 39 | 52 | 65 | 78 | 91 | 104 | 117 | 130 | 143 | 156 | 169 | 182 | 195 | 208 |
| 14 | 14 | 28 | 42 | 56 | 70 | 84 | 98 | 112 | 126 | 140 | 154 | 168 | 182 | 196 | 210 | 224 |
| 15 | 15 | 30 | 45 | 60 | 75 | 90 | 105 | 120 | 135 | 150 | 165 | 180 | 195 | 210 | 225 | 240 |
| 16 | 16 | 32 | 48 | 64 | 80 | 96 | 112 | 128 | 144 | 160 | 176 | 192 | 208 | 224 | 240 | 256 |

By Grade 5   By Grade 6   By Grade 7   By Grade 8   © ABCron

**Left Col. toward Right Col. →**

| Number | Square | Cube |
|---|---|---|
| $x$ | $x^2$ | $x^3$ |
| 1 | 1 | 1 |
| 2 | 4 | 8 |
| 3 | 9 | 27 |
| 4 | 16 | 64 |
| 5 | 25 | 125 |
| 6 | 36 | 216 |
| 7 | 49 | 343 |
| 8 | 64 | 512 |
| 9 | 81 | 729 |
| 10 | 100 | 1000 |
| 11 | 121 | 1331 |
| 12 | 144 | 1728 |
| 13 | 169 | 2197 |
| 14 | 196 | 2744 |
| 15 | 225 | 3375 |
| 16 | 256 | 4096 |
| 17 | 289 | 4913 |
| 18 | 324 | 5832 |
| 19 | 361 | 6859 |
| 20 | 400 | 8000 |
| 21 | 441 | 9261 |
| 22 | 484 | 10648 |
| 23 | 529 | 12167 |
| 24 | 576 | 13824 |
| 25 | 625 | 15625 |

**← Roots: Right Col. To Number Col.**

**Square:**
$6^2 = 36$

**Square Root:**
$\sqrt{36} = 6$

**Cube:**
$6^3 = 216$

**Cube Root:**
$\sqrt[3]{216} = 6$

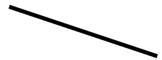
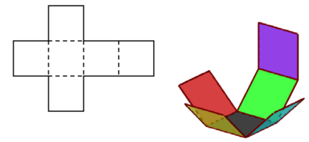
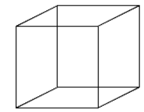
Learning the table above may seem difficult initially, but if one is willing to make the effort a little at a time, one can learn them well. One of my 7th grade teachers told us that learn them to the 16s will make our lives easier to succeed in future endeavors. The GED recommends this level of mathematical knowledge. Working on them daily helps.

**GED® TEST Tip (Kaplan © p. 320)**

**Make a list of common square roots by squaring the numbers from 1 to 1 5. Memorize them. You will need to know common square roots to solve geometry problems about area and right triangles. (I recommend the Table on this page.)**

# MATH REFERENCE PAGES

**Location, Perimeter, Area, Surface Area, and Volume**

| **Questions** | **Location** | **Perimeter** | **Area** | **Surface Area** | **Volume** |
|---|---|---|---|---|---|
| What is ______? | A point is space or in a plane. | Length of a line or a path around 2D shape | Number of squares covering a 2D shape | Number of squares covering a 3D shape | Number of cubes filling a 3D shape |
| Graphic |  |  |  |  |  |
| Dimensions ___? | 0 — D | 1—D | 2—D | 2—D | 3—D |
| Shape? | None | Line around a shape | Number of squares covering a 2D shape | Number of squares covering a 3D shape | Number of cubes filling a 3D shape |
| Units | None | in, ft, yards, miles, cm, dm, m, km | $in^2$, $ft^2$, $yds^2$, $miles^2$, $cm^2$, $dm^2$, $m^2$, $km^2$ | $in^2$, $ft^2$, $yds^2$, $miles^2$, $cm^2$, $dm^2$, $m^2$, $km^2$ | $in^3$, $ft^3$, $yds^3$, $miles^3$, $cm^3$, $dm^3$, $m^3$, $km^3$ |
| Measurement type | None | Linear | Square | Square | Cubic |
| Example of use | Location | String length | Wrapping paper squares | Wrapping paper squares | Water in pool,<br>Dirt in garden bed<br>Cubes |
| Arithmetic used | Coordinates | Add | Multiply 2 dimensions | Multiply 2 dimensions | Multiply 3 dimensions |
| Formulas | None | Add all sides<br>$C = \pi d$ circle<br>$C = 2\pi r$ circle | $A = bh$ parallelogram<br>$A = lw$ rectangle<br>$A = s^2$ square<br>$A = \frac{1}{2} bh$ triangle<br>$A = \pi r^2$ circle | $SA = 2lw + 2lh + 2wh$<br>$SA = 2\pi rh + 2\pi r^2$ cylinder<br>$SA = \pi rs + \pi r^2$ cone<br>$SA = 4\pi r^3$ sphere<br>$SA = \frac{1}{2}ps + B$ pyramid | $V = lwh$ Parallelepiped<br>$V = \frac{4}{3}\pi r^3$ Sphere<br>$V = \frac{1}{3}\pi r^2 h$ Cone<br>$V = \pi r^2 h$ Cylinder<br>$V = \frac{1}{3}Bh$ pyramid |

**Note**: All area and volume formulas are for simple shapes. For compound shapes, parts may need to be dropped from the formula. Refer to the explanations in GED Formulas Explained.

https://www.geogebra.org/m/QAPeq2cw Nets of a Cube Interactive

Read **Flatland: A Romance of Many Dimensions by Edwin A. Abbott** http://www.geom.uiuc.edu/~banchoff/Flatland/