Applied Graph Theory in the School Mathematics Curriculum

Šárka Voráčová
Department of Applied Mathematics, Faculty of Transportation, CTU in Prague

Let the main object... to seek and to find a method of instruction, by which teachers may teach less, but learners learn more.

Jan Amos Komenský
Why Graph theory?

- **Beauty and utility**
  - Valuable mathematical modeling tool. Close relations between mathematics and the real world.
  - Applicability in a wide range of areas: molecular structure, genetics, taxonomy, cartography, mind mapping, logistics, crystallography, information technology.
Why Graph theory?

- Basics are intuitive × advance topics of various difficulty.
- Lots of interesting problems accessible for 7-12 grades.
- Graphs may require a different way of thinking – chance for poor skilled students.

Graph Theory in Schoolbooks

Hubert never ride the same trial twice. Fill in the missed intersection.

stated that in grades 9-12, the mathematics curriculum should include topics from discrete mathematics so that all students can

- represent problem situations using discrete structures such as finite graphs, matrices, sequences, and recurrence relations;
- develop and analyze algorithms

At the right is a digraph concerning the diplomatic relations among certain governments.

9. How would the USSR communicate with Madagascar?

10. How would Taiwan communicate with China?

11. Which countries are most isolated?

Which country has the most diplomatic relations?
Graph is an abstract structure to represent relations between pairs of objects.
Graph Isomorphism

One graph has many different drawings - drawing is nothing more than a visualization.

Determining whether two finite graphs are isomorphic is not known to be solvable in polynomial time.
Schlegel Diagram
Planar Graphs of Platonic Solids

Schlegel diagram is a projection of a polytope through a point beyond one of its facets or faces.

Luca Pacioli: *De divina proportione*, 1509, illustrated by Leonardo da Vinci

Š. Voráčová: *Graph Theory for Kids*
Schlegel Diagram
Planar Graphs of Platonic Solids

Tetrahedron
Cube
Octahedron
Dodecahedron
Icosahedron
Planar Graphs of Archimedean Solids

- **Truncated octahedron**
- **Truncated tetrahedron**
- **Fullerene C_{60}**

Robert Webb's Stella software
A graph is a collection of vertices joined by edges.

This graph has 5 vertices, 7 edges, and it divides the plane into 4 regions.

How about this graph?

In this graph, the diagonal edges cross. Let's fix that.

Add a vertex or move an edge.

Now it works! A planar graph can be drawn without edges crossing.
Perfect Maze and the Tree

Mazes containing no loops are equivalent to a tree.

Perfect maze: all its walls are connected together or to the maze's outer boundary.
Wall follower

By keeping one hand in contact with one wall of the maze the solver is guaranteed not to get lost and will reach a different exit.
Graph Coloring

The most popular topics

Four color theorem
given any separation of a plane into contiguous regions, no more than four colors are required to color the regions of the map so that no two adjacent regions have the same color.

Two regions are called adjacent if they share a common boundary that is not a corner.

1\textsuperscript{st} proof:
1976 by Kenneth Appel and Wolfgang Haken.

- Draw a connected graph that has five vertices and a chromatic number of four.
- Draw a connected graph that has five vertices and a chromatic number of two.

GeoGebra: Vertex Coloring

There are 0 vertices without color.
Coloring algorithm

There is no known efficient algorithm for coloring any graph with the fewest number of colors possible.

**Welsh and Powell algorithm**: Sort vertices by a degree.
1. Color the highest uncolored vertex with an unused color.
2. Go down the list coloring as many uncolored vertices with the current color as you can.
3. If all the vertices are now colored, you are finished. If not, go back to Step 1.
Seven new radio stations are planning to start broadcasting in the same region of the country. It is to assign a frequency to each station so that no two stations interfere with each other.

What is the fewest possible number of new frequencies. Stations within 500 miles of each other must be assigned different frequencies. The location is given by map.
Euler circuit
Fleury’s algorithm (1883)

At each step it chooses the next edge in the path to be one whose deletion would not disconnect the subgraph of remaining edges.

It then moves to the other endpoint of that edge and deletes the edge.
Hopkins, B., R. J. Wilson, R. J. *The Truth about Königsberg*, 2004
Chinese postman problem on FTS, CTU in Prague

B. Vlček: Transportation services for a Particular Part of a Network, (Czech), bachelor thesis FTS, CTU in Prague, 2015

Optimization of the current plan of mechanical maintenance of the large network of cross-country ski trails in Jizera Mountains with the help of the graph theory methods for solving the route inspection problem. Total length 235 km, 3 snowcats Average costs 430 Kč/km
Weights are either constant or are determined by the Euclidean distance between connected vertices, that is, by the length of the edges.

1959, Edsger W. Dijkstra
An algorithm that solves the shortest path problem for connected and weighted graphs.
R. M. Falcón, A. Moreno, R. Ríos: **Designing evacuations routes with GeoGebra**

Dynamic evacuation map that make possible a fast update of the evacuation routes of a building.

*GeoGebra: Algoritmo de Dijkstra*
TravellingSalesman[towns]

Finding the tour or cyclic path of minimum length that passes through all the points exactly once.
Travelling Salesman Problem (TSP) on FTS CTU in Prague

D. Mocková: Using of Graph Theory for Summoning Flood of members odf the Commission, Perners Contacts, No 8, 2013

Coupling paths for ensuring accessibility members of the commission on flood alert issued by the Czech Hydro-meteorological Institute and the announcement of the first degree flood flow, which cannot use the services of the mobile operator.
VRP (Vehicle Routing Problem)

- "What is the optimal set of routes for a fleet of vehicles to traverse in order to deliver to a given set of customers?"
- Combinatorial optimization and integer programming problem.
- Manual vs. Automatic Optimum Routing (percentage difference is on average 15%)
Vehicle Routing Problem on FTS, CTU in Prague

Logistics and supply chain optimization (Czech)
The use of computer optimization programs can give savings of 10%.

Rybičková: Analýza a optimalizace zásobování autorizovaných servisů vozidel Peugeot a Citroën, Faculty of Transportation CTU, 2010, 2012.

Analysis and optimization of spare parts distribution to the authorized repair network of Peugeot and Citroen in the Czech Republic, the use of genetic algorithms in transportation-optimization problems

Clark-Wright Method
Total length 2495 km.

Constraints: One route max 400 km, max 8 services.