Circular Functions

\[ y \]

\[ \begin{array}{c}
\sin \theta = \frac{1}{\cos \theta} \\
\cos \theta = \frac{1}{\sin \theta}
\end{array} \]

Hypotenuse

Adjacent

\[ \theta \]

Name:
Circular Functions (Trigonometry)

Circular functions Revision

Where do $\sin \theta, \cos \theta$ and $\tan \theta$ come from?
Unit circle (of radius 1)

- $\cos \theta$ is the $x$ – coordinate
- $\sin \theta$ is the $y$ – coordinate
- $\tan \theta = \frac{\sin \theta}{\cos \theta}$
- all 3 are measures of length.
- Remember SOH CAH TOA
- Exact values:

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>0</th>
<th>$\frac{30^\circ}{6}$</th>
<th>$\frac{45^\circ}{4}$</th>
<th>$\frac{60^\circ}{3}$</th>
<th>$\frac{90^\circ}{2}$</th>
<th>$\frac{180^\circ}{\pi}$</th>
<th>$\frac{270^\circ}{3\pi}$</th>
<th>$\frac{360^\circ}{2\pi}$</th>
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<tbody>
<tr>
<td>$\cos \theta$</td>
<td>1</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$\sin \theta$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>$\tan \theta$</td>
<td>0</td>
<td>$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$</td>
<td>1</td>
<td>$\sqrt{3}$</td>
<td>undefined</td>
<td>0</td>
<td>undefined</td>
<td>0</td>
</tr>
</tbody>
</table>

- Angle conversions (between radians and degrees).
• Quadrants and symmetry:
  o All Students Talk C.. (ASTC)

Finding Exact values:

Example: What is the exact value of:
(a) \( \sin \frac{5\pi}{4} \); (b) \( \tan \frac{-2\pi}{3} \).

(a) 1. Sign: 3\(^{rd}\) Quadrant \(\Rightarrow\) -ve
   2. Angle Equivalent (1\(^{st}\) Quadrant): \( \frac{5\pi}{4} = \pi + \frac{\pi}{4} \Rightarrow \frac{\pi}{4} \)
   3. So: \( \sin \frac{5\pi}{4} = -\sin \frac{\pi}{4} = -\frac{1}{\sqrt{2}} \) or \( -\frac{\sqrt{2}}{2} \)

(b) 1. Sign: 2\(^{nd}\) “negative” Quadrant \(\Rightarrow\) +ve
   2. Angle Equivalent (1\(^{st}\) Quadrant): \( \frac{-2\pi}{3} = -\pi + \frac{\pi}{3} \Rightarrow \frac{\pi}{3} \)
   3. So: \( \tan \frac{-2\pi}{3} = \tan \frac{\pi}{3} = \sqrt{3} \)

Jump Start Holiday Questions

**Review:** radians, definitions, exact values, symmetry

| Ex6A Q 1, 2, 3, 4 (ace for all); Ex6B Q 1, 2acegik, 3 acegikmoqsu, 4 aceg, 5 abdfgj, 6 Ex6C Q 2 |

*CALCULATOR MODE: Always work in radians*
Solving equations involving circular functions.

Finding axis intercepts:
1. Y-intercepts:
   - \( f(0) \) or \( x = 0 \).
   - E.g. what is the Y-intercept of \( f(x) = 3\sin 2\left( x - \frac{\pi}{6} \right) + 2 \)

2. X-intercepts:
   - \( f(x) = 0 \) or \( y = 0 \).

Examples: Find all values of \( \theta \) for:
(a) \( \\{ \theta : \cos \theta = \frac{\sqrt{3}}{2}, \ \theta \in [0, 2\pi] \} \)

(b) \( \{ \theta : \sin \theta = -0.7, \ \theta \in [0, 2\pi] \} \)
(c) \[ \theta : 2\sin \theta + 1 = 0, \quad \theta \in [-2\pi, 2\pi] \]

(d) \[ 4\cos 2\theta + 2 = 0, \quad \theta \in [0, 2\pi] \]

- Ex6E 1 ace, 2 ac, 3 ac, 4 ab, 5 abc, 6 ace, 7 ace, 8 acegi; Ex6J 4, 5, 6

Using the TI-Nspire

Use Solve from the Algebra menu as shown.

Using the TI-Nspire

To find the x-axis intercepts, Enter
\[ \text{solve}(3 \tan \left(2x - \frac{\pi}{3}\right) = -\sqrt{3}, x) \quad \frac{\pi}{6} \leq x \leq \frac{13\pi}{6} \]

2011 Exam1
Graphs of Circular Functions

\[ y = \sin \theta \]

- Period = \(2\pi\)
- Amplitude = 1
- Range: \([-1, 1]\)

\[ y = \cos \theta \]

\[ y = \tan \theta \]

- Period = \(\pi\)
- We don’t refer to the amplitude for \(y = \tan \theta\)
- Range: \(R\)
Transformations of $y = \sin \theta$ & $y = \cos \theta$

- $a$: a dilation of factor “$a$” from the $x$-axis.
- $n$: a dilation of factor “$\frac{1}{n}$” from the $y$-axis.
- $b$: a translation of $b$ units along the $x$-axis.
- $c$: a translation of $c$ units along the $y$-axis.

1. Dilations
   (a) The effect of “$a$”

Graph the following graphs: (i) $y = 2\cos \theta$; (ii) $y = \frac{\sin \theta}{2}$; where $\theta \in [0, 2\pi]$

- “$a$” affects the amplitude.

(b) The effect of “$n$”

Graph the following graphs: (i) $y = 3\cos 2\theta$; (ii) $y = 3\sin \left(\frac{\theta}{2}\right)$; where $\theta \in [0, 2\pi]$

- “$n$” affects the period.
- $\text{period} = \frac{2\pi}{n}$
2. Reflections.
- Two types:
  - Reflection in the $x$-axis: $-f(x)$
  - Reflection in the $y$-axis: $f(-x)$

Examples:
Sketch the graphs of the following:
(a) $y = -3 \sin 2\theta$; (b) $y = 2 \cos \left(-\frac{\pi \theta}{3}\right)$; where $\theta \in [0, \ 2\pi]$

3. Translations
(a) The effect of “$c$”
Sketch the following: (i) $y = 3 \sin \theta + 3$; (ii) $y = 2 \cos 2\theta - 3$; where $\theta \in [0, \ 2\pi]$
(i) 
(ii)
(b) The effect of “b”
Sketch the following:
(i) \( y = 2\sin\left(\theta + \frac{\pi}{4}\right) \); (ii) \( y = 3\cos\left(\theta - \frac{\pi}{3}\right) \); where \( \theta \in [0, 2\pi] \)

Combining all transformations
Example: Sketch the graph of \( f(\theta) = 3\sin\left(2\theta - \frac{\pi}{2}\right) + 2 \), \( \theta \in [0, 2\pi] \)
Rewrite: \( f(\theta) = 3\sin\left(2\left(\theta - \frac{\pi}{4}\right)\right) + 2 \)
\( a = 3, b = \frac{\pi}{4}, c = 2 \) and \( n = 2 \)
Sketch \( f(\theta) = 3\sin 2\theta \) first:
Secondly with translations:

Note: X-intercepts need to be found!!
- Ex6F 1 adfhi, 2, 4, 5; Ex6G 1, 2 ac, 3 ef, 5 acfgh, 6, 7
Graphs & Transformations of the Tangent function

Example: Sketch $y = 3\tan\left(2x - \frac{\pi}{3}\right)$ for $\frac{\pi}{6} \leq x \leq \frac{13\pi}{6}$

Rewrite: $y = 3\tan \left(2\left(x - \frac{\pi}{6}\right)\right)$

Ex6J 1, 2, 7, 8, 9
Addition of ordinates (add the ‘y’ values)

Example:

(a) On the same set of axes sketch \( f(x) = 2\sin x \) and \( g(x) = 3\cos 2x \) for \( 0 \leq x \leq 2\pi \);
(b) Use addition of ordinates to sketch the graph of \( y = 2\sin x + 3\cos 2x \).

Note: For \( y = 2\sin(x) - 3\cos(2x) \) it is easier to do \( y = 2\sin(x) + (-3\cos(2x)) \)

- Ex6H 1 ace
Solving Equations where both $\sin$ & $\cos$ appear

Example: Solve for $x$, $x \in [0, 2\pi]$:

(i) $\sin x = 0.5 \cos x$

(ii) $\sin 3x - \sqrt{3} \cos 3x = 0$

- Ex6J 10, 11 acegi, 12
General Solutions to Circular Functions

Example: Solve \( \cos x = \frac{1}{2} \)

\[
\cos x = \frac{1}{2}
\]

1. Cos positive Quad ...
2. Angle :
3. \( x = \ldots \)

Solution:
\[
x = \ldots,
\]

generally:
\[
x = \quad \text{Check: } n = 0, n = 1, n = -1
\]

So in general terms:

Example: Solve \( \sin x = \frac{1}{2} \)

\[
\sin x = \frac{1}{2}
\]

1. Sin positive Quad ...
2. Angle :
3. \( x = \ldots \)

Solution:
\[
x = \ldots,
\]

generally:
\[
x = \quad \text{Check: } n = 0, n = 1, n = -1
\]

or
\[
x = \quad \text{Check: } n = 0, n = 1, n = -1
\]

So in general terms:

The above can be simplified to

\[
\tan x = a \quad \Rightarrow \quad x = n\pi + \tan^{-1}(a), \quad n \in \mathbb{Z}
\]
Example 1: Find the general solution for \( 2 \sin \left( x + \frac{\pi}{3} \right) = -1 \)

Solution:

Example 2: Find the general solution to \( 2 \cos \left( 2x + \frac{\pi}{4} \right) = \sqrt{2} \), and hence find all the solutions from \((-2\pi, 2\pi)\).

Solution:
Using the TI-Nspire

Make sure the calculator is in Radian mode.

a  Use Solve from the Algebra menu and complete as shown.
   Note the use of \( \frac{1}{2} \) rather than 0.5 to ensure that the answer is exact.

b  Complete as shown.

c  Complete as shown.
Determining Rules for Circular Functions

Example: The graph shown has the rule of the form: \( y = a \cos(n(t - b)) + c \), find \( a, b, c \) & \( n \).

\[
\begin{align*}
\text{Ex6I} & \quad 1, 2, 4, 5, 6, 7, 8, 9; \quad \text{Ex6J} \quad 14, 15
\end{align*}
\]
Applications of Circular Functions

**worked example 24**

The temperature in degrees Celsius on a day in May at Mt Buller is expected to follow the model

\[ T = 5 - 7 \cos \left( \frac{\pi}{12} (t - 4) \right) \]

where \( t \) is the number of hours after midnight. The snow-making machines will only operate efficiently when the temperature is below 5°C. Sketch the graph of the temperature for one full day, and predict the period of time for which the machine will be able to operate.

- Ex6L 1, 2, 4, 6 Ex 6N
Past Exam Questions

2008

**Question 3**
Solve the equation \( \cos \left( \frac{3x}{2} \right) = \frac{1}{2} \) for \( x \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \).

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2 marks

**Question 18**
Let \( f: \left[ 0, \frac{\pi}{2} \right] \to \mathbb{R}, f(x) = \sin(4x) + 1 \). The graph of \( f \) is transformed by a reflection in the x-axis followed by a dilation of factor 4 from the y-axis.

The resulting graph is defined by

A. \( g: \left[ 0, \frac{\pi}{2} \right] \to \mathbb{R}, g(x) = -1 - 4 \sin(4x) \)

B. \( g: [0, 2\pi] \to \mathbb{R}, g(x) = -1 - \sin(16x) \)

C. \( g: \left[ 0, \frac{\pi}{2} \right] \to \mathbb{R}, g(x) = 1 - \sin(x) \)

D. \( g: [0, 2\pi] \to \mathbb{R}, g(x) = 1 - \sin(4x) \)

E. \( g: [0, 2\pi] \to \mathbb{R}, g(x) = -1 - \sin(x) \)

2009

**Question 4**
Solve the equation \( \tan(2x) = \sqrt{3} \) for \( x \in \left\{ -\frac{\pi}{4}, \frac{\pi}{4} \right\} \cup \left\{ \frac{\pi}{4}, \frac{3\pi}{4} \right\} \).

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3 marks
Question 4
The general solution to the equation \( \sin(2x) = -1 \) is

A. \( x = n\pi - \frac{\pi}{4}, \ n \in Z \)
B. \( x = 2n\pi + \frac{\pi}{4} \) or \( x = 2n\pi - \frac{\pi}{4}, \ n \in Z \)
C. \( x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{2}, \ n \in Z \)
D. \( x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{4}, \ n \in Z \)
E. \( x = n\pi + \frac{\pi}{4} \) or \( x = 2n\pi + \frac{\pi}{4}, \ n \in Z \)

Question 12
A transformation \( T: \mathbb{R}^2 \rightarrow \mathbb{R}^3 \) that maps the curve with equation \( y = \sin(x) \) onto the curve with equation \( y = 1 - 3 \sin(2x + \pi) \) is given by

A. \( T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \pi \\ 1 \end{bmatrix} \)
B. \( T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \pi \\ 1 \end{bmatrix} \)
C. \( T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} \pi \\ 1 \end{bmatrix} \)
D. \( T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\frac{\pi}{2} \\ 1 \end{bmatrix} \)
E. \( T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\frac{\pi}{2} \\ -1 \end{bmatrix} \)
Question 4

a. Write down the amplitude and period of the function

\[ f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 4 \sin \left( \frac{x + \pi}{3} \right). \]

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2 marks

b. Solve the equation \( \sqrt{3} \sin(x) = \cos(x) \) for \( x \in [-\pi, \pi] \).

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2 marks

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Question 3

An ancient civilisation buried its kings and queens in tombs in the shape of a square-based pyramid, \( WABCD \).

The kings and queens were each buried in a pyramid with \( WA = WB = WC = WD = 10 \) m.

Each of the isosceles triangle faces is congruent to each of the other triangular faces.

The base angle of each of these triangles is \( x \), where \( \frac{\pi}{4} < x < \frac{\pi}{2} \).

Pyramid \( WABCD \) and a face of the pyramid, \( WAB \), are shown here.

---

\[ Z \] is the midpoint of \( AB \).

a. i. Find \( AB \) in terms of \( x \).
b. Show that the total surface area (including the base), \( S \text{ m}^2 \), of the pyramid, \( WABCD \), is given by
\[
S = 400(\cos^2(\alpha) + \cos(\gamma) \sin(\gamma))
\]

2 marks

c. Find \( WT \), the height of the pyramid \( WABCD \), in terms of \( x \).

2 marks

d. The volume of any pyramid is given by the formula
\[
\text{Volume} = \frac{1}{3} \times \text{area of base} \times \text{vertical height}
\]

Show that the volume, \( T \text{ m}^3 \), of the pyramid \( WABCD \) is
\[
T = \frac{4000}{3} \sqrt{\cos^4 x - 2 \cos^6 x}
\]
2011

Question 3
a. State the range and period of the function
\[ h: \mathbb{R} \rightarrow \mathbb{R}, \ h(x) = 4 + 3\cos\left(\frac{\pi x}{2}\right). \]
-------------------------------------------------------------------------------------

b. Solve the equation
\[ \sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2} \text{ for } x \in [0, \pi]. \]
-------------------------------------------------------------------------------------

Question 10
The figure shown represents a wire frame where \(ABCE\) is a convex quadrilateral. The point \(D\) is on line segment \(EC\) with \(AB = ED = 2\) cm and \(BC = a\) cm, where \(a\) is a positive constant.
\[ \angle BAE = \angle CEA = \frac{\pi}{2} \]
Let \(\angle CBD = \theta\) where \(0 < \theta < \frac{\pi}{2}\).

![Diagram showing a wire frame with points A, B, C, D, and E, with angles and lengths labeled.]  

a. Find \(BD\) and \(CD\) in terms of \(a\) and \(\theta\).
-------------------------------------------------------------------------------------
Question 15

The graph shown could have equation

A. \( y = 2\cos\left(x + \frac{\pi}{6}\right) + 1 \)

B. \( y = 2\cos 4\left(x - \frac{\pi}{6}\right) + 1 \)

C. \( y = 4\sin 2\left(x - \frac{\pi}{12}\right) - 1 \)

D. \( y = 3\cos\left(2x + \frac{\pi}{6}\right) - 1 \)

E. \( y = 2\sin\left(4x + \frac{2\pi}{3}\right) - 1 \)

2012

Question 6

The graphs of \( y = \cos(x) \) and \( y = a \sin(x) \), where \( a \) is a real constant, have a point of intersection at \( x = \frac{\pi}{3} \).

a. Find the value of \( a \).

2 marks

b. If \( x \in [0, 2\pi] \), find the \( x \)-coordinate of the other point of intersection of the two graphs.

1 mark
Question 1
The function with rule \( f(x) = -3 \sin\left(\frac{\pi x}{5}\right) \) has period
A. 3
B. 5
C. 10
D. \( \frac{\pi}{5} \)
E. \( \frac{\pi}{10} \)

Question 6
A section of the graph of \( f \) is shown below.

The rule of \( f \) could be
A. \( f(x) = \tan x \)
B. \( f(x) = \tan \left( x - \frac{\pi}{4} \right) \)
C. \( f(x) = \tan \left( 2 \left( x - \frac{\pi}{4} \right) \right) \)
D. \( f(x) = \tan \left( 2 \left( x - \frac{\pi}{2} \right) \right) \)
E. \( f(x) = \tan \left( \frac{1}{2} \left( x - \frac{\pi}{4} \right) \right) \)
Question 7
The temperature, \( T \) °C, inside a building \( t \) hours after midnight is given by the function

\[
f \colon [0, 24] \to \mathbb{R}, \quad T(t) = 22 - 10 \cos \left( \frac{\pi}{12} (t - 2) \right)
\]

The average temperature inside the building between 2 am and 2 pm is

A. 10 °C  
B. 12 °C  
C. 20 °C  
D. 22 °C  
E. 32 °C

Question 19
A function \( f \) has the following two properties for all real values of \( \theta \).

\[
f(\pi - \theta) = -f(\theta) \quad \text{and} \quad f(\pi + \theta) = -f(-\theta)
\]

A possible rule for \( f \) is

A. \( f(x) = \sin(x) \)  
B. \( f(x) = \cos(x) \)  
C. \( f(x) = \tan(x) \)  
D. \( f(x) = \sin \left( \frac{x}{2} \right) \)  
E. \( f(x) = \tan(2x) \)

2013

Question 4 (2 marks)
Solve the equation \( \sin \left( \frac{x}{2} \right) = -\frac{1}{2} \) for \( x \in [2\pi, 4\pi] \).

\[
\text{----------------------------------------}
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\[
\text{----------------------------------------}
\]
Question 1
The function with rule \( f(x) = -3 \tan(2\pi x) \) has period

A. \( \frac{2}{\pi} \)

B. 2

C. \( \frac{1}{2} \)

D. \( \frac{1}{4} \)

E. \( 2\pi \)

Question 7
The function \( g: [-\alpha, \alpha] \to \mathbb{R}, \ g(x) = \sin \left( 2 \left( x - \frac{\pi}{6} \right) \right) \) has an inverse function. The maximum possible value of \( \alpha \) is

A. \( \frac{\pi}{12} \)

B. 1

C. \( \frac{\pi}{6} \)

D. \( \frac{\pi}{4} \)

E. \( \frac{\pi}{2} \)
Question 1 (12 marks)

Trigg the gardener is working in a temperature-controlled greenhouse. During a particular 24-hour time interval, the temperature ($T$ °C) is given by $T(t) = 25 + 2\cos\left(\frac{\pi t}{8}\right)$, $0 \leq t \leq 24$, where $t$ is the time in hours from the beginning of the 24-hour time interval.

a. State the maximum temperature in the greenhouse and the values of $t$ when this occurs. 2 marks

b. State the period of the function $T$. 1 mark

c. Find the smallest value of $t$ for which $T = 26$. 2 marks

d. For how many hours during the 24-hour time interval is $T \geq 26$? 2 marks
Question 1 (7 marks)
The population of wombats in a particular location varies according to the rule
\[ n(t) = 1200 + 400 \cos \left( \frac{\pi t}{3} \right), \]
where \( n \) is the number of wombats and \( t \) is the number of months after 1 March 2013.

a. Find the period and amplitude of the function \( n \).  

b. Find the maximum and minimum populations of wombats in this location.  

c. Find \( n(10) \).  

d. Over the 12 months from 1 March 2013, find the fraction of time when the population of wombats in this location was less than \( n(10) \).  

Question 3 (2 marks)
Solve \( 2 \cos(2x) = -\sqrt{3} \) for \( x \), where \( 0 \leq x \leq \pi \).
Question 5 (3 marks)
On any given day, the depth of water in a river is modelled by the function
\[ h(t) = 14 + 8 \sin \left( \frac{\pi t}{12} \right) \quad 0 \leq t \leq 24 \]
where \( h \) is the depth of water, in metres, and \( t \) is the time, in hours, after 6 am.
a. Find the minimum depth of the water in the river. 1 mark

b. Find the values of \( t \) for which \( h(t) = 10 \). 2 marks

Question 10 (7 marks)
The diagram below shows a point, \( T \), on a circle. The circle has radius 2 and centre at the point \( C \) with coordinates \((2, 0)\). The angle \( ECT \) is \( \theta \), where \( 0 < \theta \leq \frac{\pi}{2} \).

![Diagram showing a circle with points \( C \), \( E \), \( X \), \( D(4, d) \), \( B(2, b) \), and \( T \) on the circumference.]

The diagram also shows the tangent to the circle at \( T \). This tangent is perpendicular to \( CT \) and intersects the x-axis at point \( X \) and the y-axis at point \( Y \).
a. Find the coordinates of \( T \) in terms of \( \theta \). 1 mark

b. Find the coordinates of \( T \) in terms of \( \theta \). 2 marks

c. Find the coordinates of \( X \) in terms of \( \theta \). 2 marks

d. Find the coordinates of \( Y \) in terms of \( \theta \). 2 marks

2015
b. Find the gradient of the tangent to the circle at $T$ in terms of $\theta$. 1 mark

\[
\cos(\theta)x + \sin(\theta)y = 2 + 2\cos(\theta)
\]

\[\text{\text{ }}\]

\[\text{\text{ }}\]

\[\text{\text{ }}\]

\[\text{\text{ }}\]

c. The equation of the tangent to the circle at $T$ can be expressed as

\[\cos(\theta)x + \sin(\theta)y = 2 + 2\cos(\theta)\]

i. Point $B$, with coordinates $(2, b)$, is on the line segment $XT$.

Find $b$ in terms of $\theta$. 1 mark

\[\text{\text{ }}\]

\[\text{\text{ }}\]

\[\text{\text{ }}\]

\[\text{\text{ }}\]

\[\text{\text{ }}\]

ii. Point $D$, with coordinates $(4, d)$, is on the line segment $XT$.

Find $d$ in terms of $\theta$. 1 mark

\[\text{\text{ }}\]

\[\text{\text{ }}\]

\[\text{\text{ }}\]

\[\text{\text{ }}\]

\[\text{\text{ }}\]

Question 1
Let $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 2\sin(3x) - 3$.
The period and range of this function are respectively

A. period $= \frac{2\pi}{3}$ and range $= [-5, -1]$

B. period $= \frac{2\pi}{3}$ and range $= [-2, 2]$

C. period $= \frac{\pi}{3}$ and range $= [-1, 5]$

D. period $= 3\pi$ and range $= [-1, 5]$

E. period $= 3\pi$ and range $= [-2, 2]$